



Mysteries of the Smith Chart

*Transmission Lines, Impedance Matching,
and Little Known Facts*

Stephen D. Stearns, K6OIK

Chief Technologist

TRW Firestorm Wireless Communication Products

stearns@ieee.org

□ *Transmission Line Theory*

- Historical development
- Heaviside's rewrite of Maxwell's theory, Telegrapher's equations,
- Impedance, reflection coefficient, SWR, phase constant, and velocity factor
- Special facts for $\lambda/2$, $\lambda/4$, and $\lambda/8$ lossless lines

□ *The Smith Chart*

- Bilinear complex functions
- Impedance and admittance coordinates (circles, circles, and more circles)

□ *Impedance Matching*

- Why match? Impedance matching vs. conjugate impedance matching
- Single frequency matching
- Multiple-frequency and broadband matching



Part 1: Transmission Line Theory

- 1830s** *Magnetic telegraphs - Gauss, Henry*
- 1839** *Electromagnetic telegraph - Wheatstone & Cook*
- 1844** *Telegraph in America - Morse*
- 1850s** *Thousands of miles of telegraph line U.S. and Europe*
- 1851** *40-mile cable under English Channel*
- 1855** *Distributed analysis of transmission line - Lord Kelvin*
- 1858** *Transatlantic cable, project delayed by civil war*
- 1873** *Theory of electrodynamics - Maxwell*
- 1876** *Invention of telephone - Bell*
- 1880s** *Vectors, vector calculus, reformulation of Maxwell's theory, transmission line theory - Heaviside*
- 1886** *Experimental confirmation of Maxwell's Theory - Hertz*
- 1937** *Early Smith Chart, published 1939 and 1944 - Smith*

Numbers to Remember!



1.4142135623...

1.7320508075...

1.6180339887...

3.1415926535...

2.718281828459045...

2.54

299,792,458

376.7303134...

Heaviside's Vector Formulation of Maxwell's Theory



$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\mathbf{D} = \epsilon \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H}$$

“And God said, Let there be light; and there was light.” Genesis 1:3

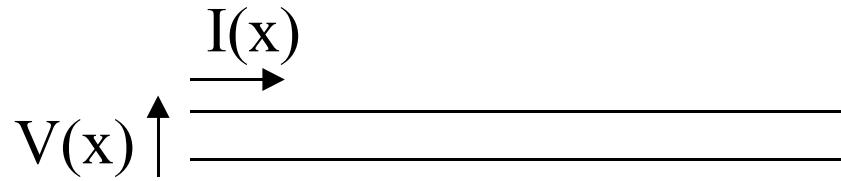
$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$

$$\nabla \times \mathbf{H} = (\sigma + j\omega\epsilon)\mathbf{E}$$

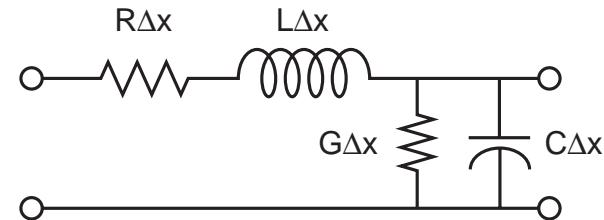
$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{H} = 0$$

Uniform transmission line



Equivalent circuit of infinitesimal segment



$$\frac{dV}{dx} = -(R + j\omega L)I(x)$$

$$\frac{dI}{dx} = -(G + j\omega C)V(x)$$

Traveling wave

$$V(x) = V_o e^{\gamma x}$$

$$I(x) = \frac{V(x)}{Z_o}$$

Propagation constant

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Characteristic impedance

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Notations

Real Parameters



R = Series resistance per unit length (Ohms/meter)

L = Series inductance per unit length (Henries/meter)

G = Shunt conductance per unit length (Siemens/meter)

C = Shunt capacitance per unit length (Farads/meter)

α = Attenuation constant (nepers/meter)

β = Phase constant (radians/meter)

λ = Wavelength (meters)

v_f = Velocity factor (dimensionless)

X = Reactance (Ohms)

B = Susceptance (Siemens)

s = Standing wave ratio (dimensionless)

Notations (Cont'd)

Complex Parameters



$Z = R + jX = \text{impedance (Ohms)}$

$Z_L = \text{Load impedance (Ohms)}$

$Z_i = \text{Input impedance (Ohms)}$

$Z_0 = \text{Characteristic impedance (Ohms)}$

$z = Z/Z_0 = r + jx = \text{normalized impedance (dimensionless)}$

$Y = G + jB = \text{admittance (Siemens)}$

$y = Y/Y_0 = g + jb = \text{normalized admittance (dimensionless)}$

$\Gamma = \Gamma_r + j\Gamma_i = \text{complex reflection coefficient (dimensionless)}$

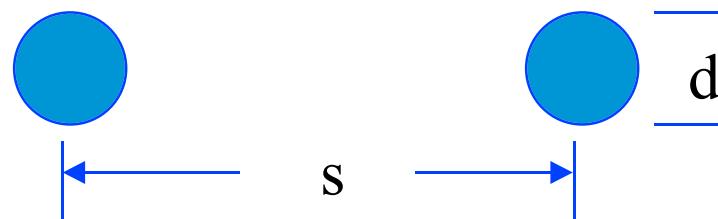
$\gamma = \alpha + j\beta = \text{propagation constant (inverse meters)}$

Transmission Line Parameters

Physical Dimensions and Material Properties



Parameter	Coax	Twinlead
R Ω/m	$\frac{1}{2\pi\delta\sigma_c} \left(\frac{1}{a} + \frac{1}{b} \right)$	$\frac{1}{\pi a \delta \sigma_c}$
L H/m	$\frac{\mu}{2\pi} \left[\ln \frac{b}{a} + \frac{\delta}{2} \left(\frac{1}{a} + \frac{1}{b} \right) \right]$	$\frac{\mu}{\pi} \left[\frac{\delta}{2a} + \cosh^{-1} \frac{d}{2a} \right]$
G S/m	$\frac{2\pi\sigma}{\ln \frac{b}{a}}$	$\frac{\pi\sigma}{\cosh^{-1} \frac{d}{2a}}$
C F/m	$\frac{2\pi\epsilon}{\ln \frac{b}{a}}$	$\frac{\pi\epsilon}{\cosh^{-1} \frac{d}{2a}}$
Where skin depth is	$\delta = \frac{1}{\sqrt{\pi f \mu \sigma_c}}$	For copper $\sigma_c = 5.8 \times 10^7 \text{ S/m}$ $\delta = \begin{cases} 8.5 \text{ mm at } 60 \text{ Hz} \\ 6.6 \mu\text{m at } 100 \text{ MHz} \end{cases}$



□ Approximate formula

- Widely published by ARRL and others
- Accurate only for large spacings: $s/d > 3$
or large impedances: $Z_0 >$ several hundred

$$Z_0 = 120 \ln \left(\frac{2s}{d} \right) = 276 \log_{10} \left(\frac{2s}{d} \right)$$

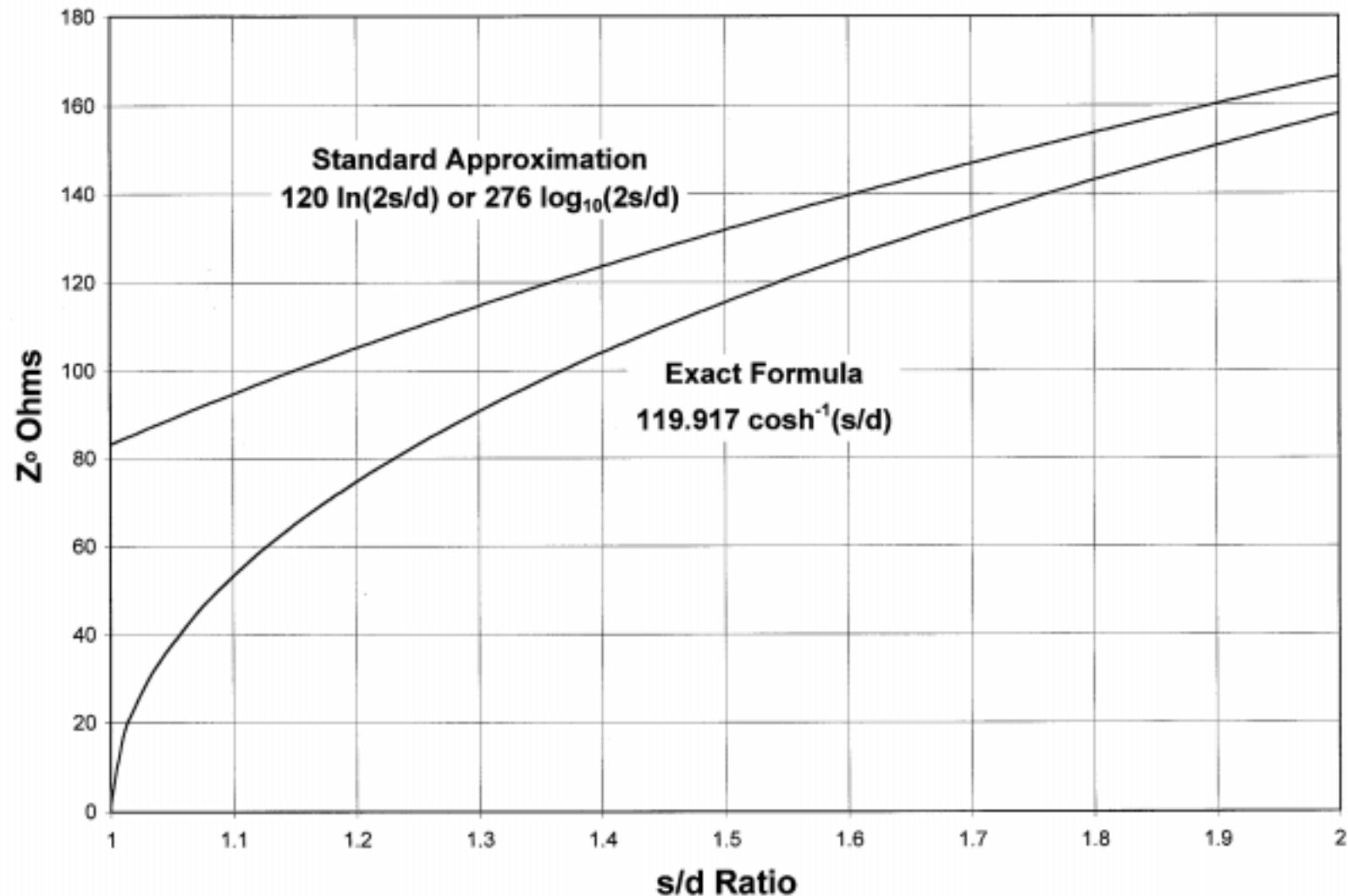
□ Exact formula

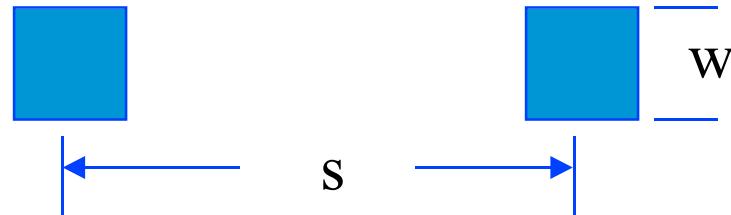
- Accurate for all spacings and impedances

$$Z_0 = 119.917 \cosh^{-1} \left(\frac{s}{d} \right)$$

Comparison of Impedance Formulas

Round Open-Wire Line





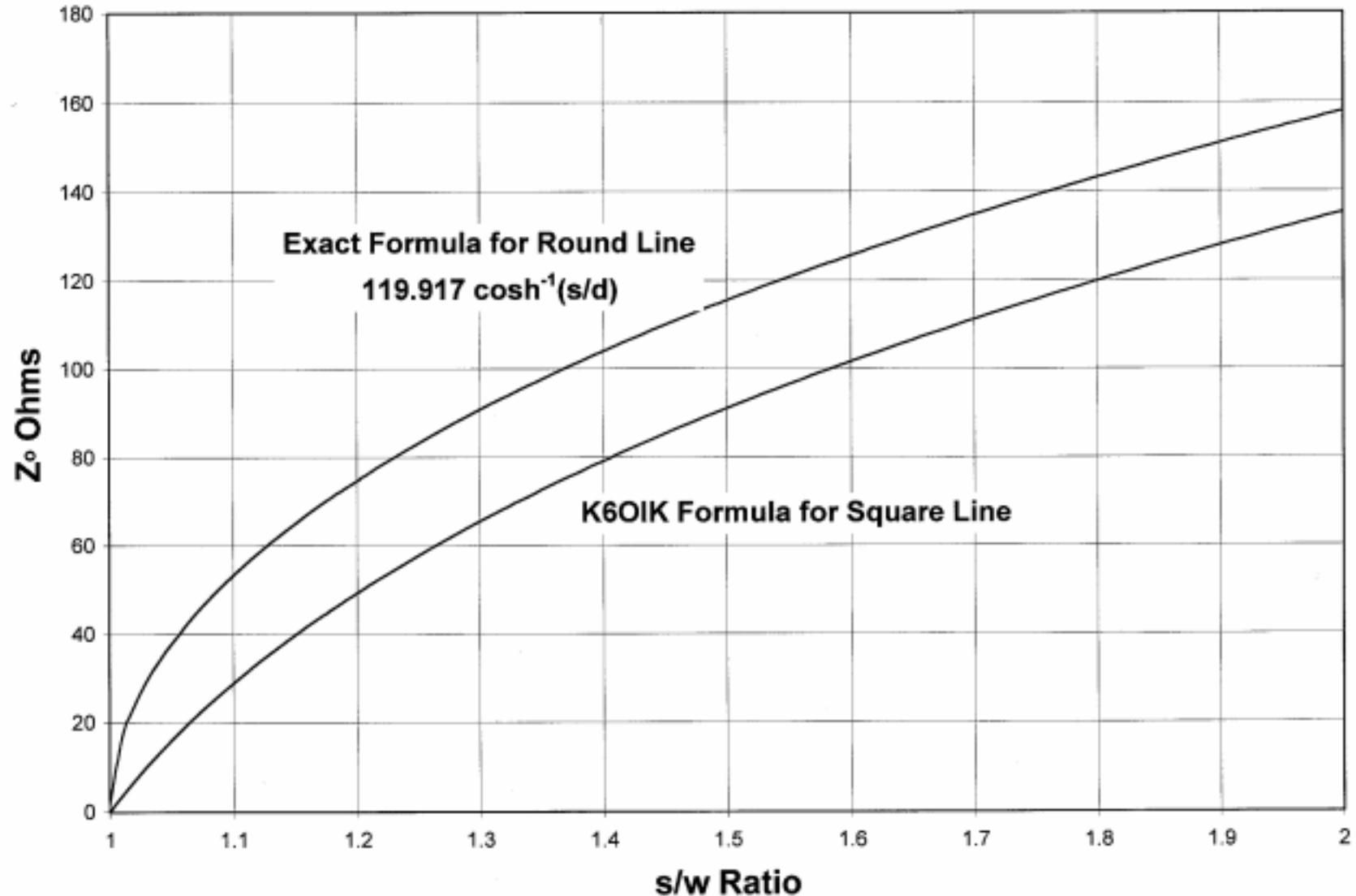
□ ***Excellent approximation in the range of practical interest***

- Accurate for small spacings: $1 < s/w < 3$
or small impedances: $0 < Z_0 <$ several hundred

$$Z_0 = \frac{376.730313461}{\frac{1}{s/w-1} + 0.483 + \frac{2}{\pi} \ln\left(\frac{s/w}{s/w-1}\right) + \frac{1}{(s/w)^{1/3}-0.1}}$$

Round vs Square Open-Wire Lines

TRW



Coax

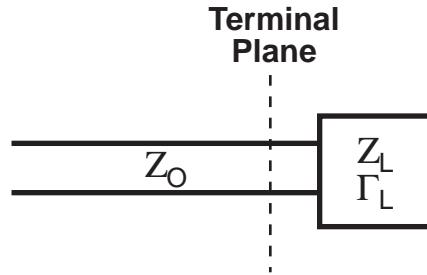
For minimum loss $Z_o = 77\Omega$

For maximum breakdown voltage $Z_o = 30\Omega$

For minimum temperature rise $Z_o = 60\Omega$

$Z_o = 50\Omega$ has no special significance

Reflection Coefficient and Impedance Relation at a Terminal Plane



Definition

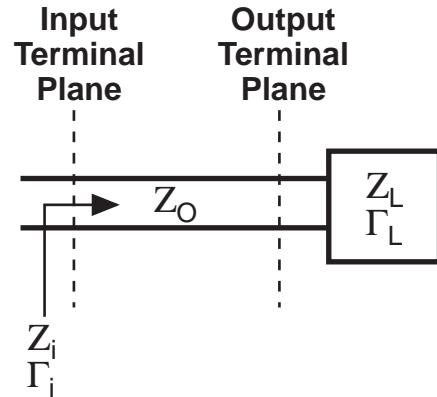
$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{z - 1}{z + 1}$$

Inverse

$$z = \frac{1 + \Gamma}{1 - \Gamma}$$

- For every terminal plane, the complex load impedance and complex reflection coefficient seen to the right give the same information for that terminal plane***
- Question: How do Γ and z change as the terminal plane moves?***

Relations Between Two Terminal Planes



Impedance relation

$$z_i = \frac{z_L + j \tan \beta l}{1 + j z_L \tan \beta l}$$

Cross relations

$$z_i = \frac{1 + \Gamma_L e^{-j2\beta l}}{1 - \Gamma_L e^{-j2\beta l}}$$

Reflection coefficient relation

$$\Gamma_i = \Gamma_L e^{-j2\beta l}$$

$$z_L = \frac{1 + \Gamma_i e^{j2\beta l}}{1 - \Gamma_i e^{j2\beta l}}$$

Wavelength

$$\lambda_{free\ space} = \frac{c}{f}$$

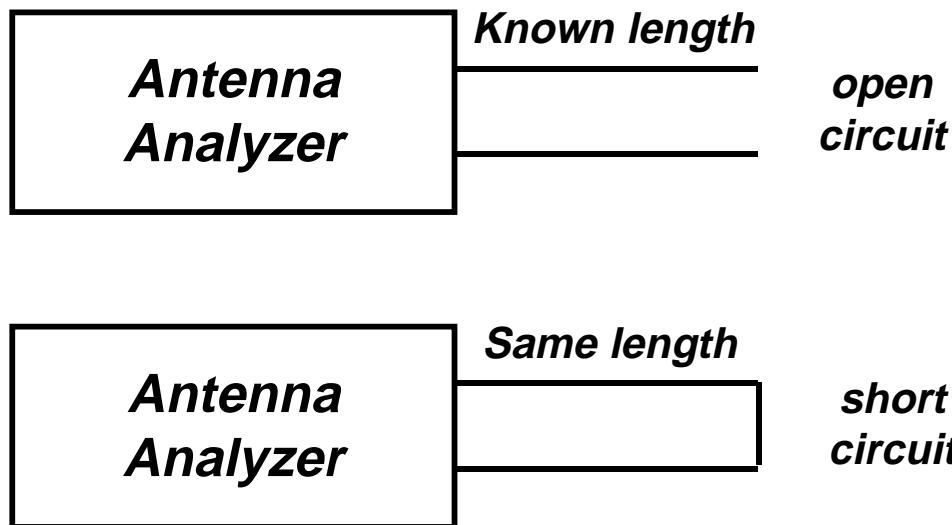
$$\lambda_{actual} = \frac{v}{f}$$

Velocity factor

$$v_f = \frac{v}{c} = \frac{\lambda_{actual}}{\lambda_{free\ space}}$$

How To Measure Velocity Factor of a Line

(One Way To Do It)



$$v_f = \frac{2\pi fl}{c} \frac{1}{\cot^{-1} \sqrt{\frac{-Z_{open}}{Z_{short}}}}$$

$$\beta = \frac{2\pi}{\lambda_{actual}} = \frac{2\pi f}{v_f c} \quad \text{radians/meter}$$

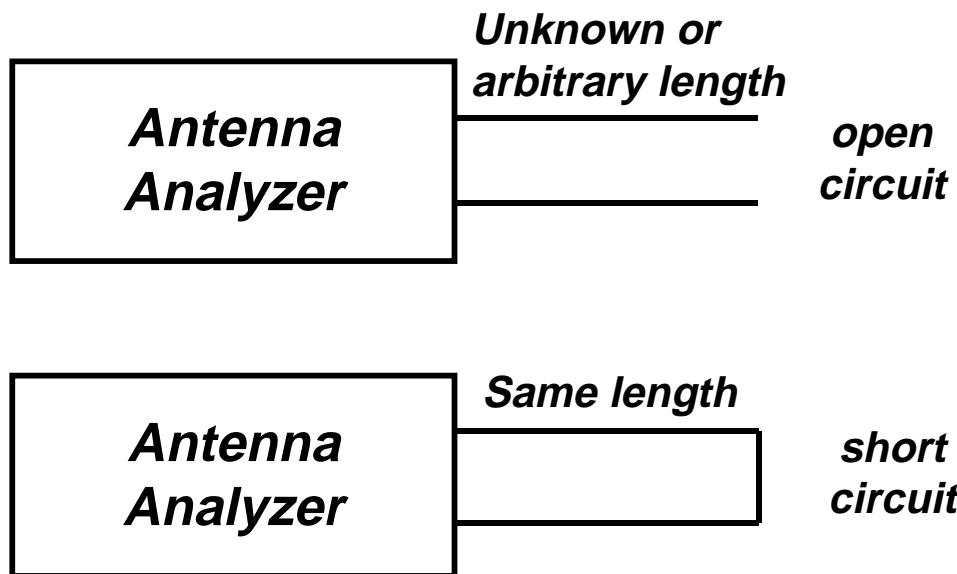
- **Phase constant β and velocity factor v_f give equivalent information**
- **Both can be calculated from line dimensions and material properties**

$$\beta = \text{Im} \sqrt{(R + j\omega L)(G + j\omega C)}$$

- **Best to measure!**

How to Measure Complex Z_o of A Line

(One Way to Do It)



$$Z_o = \sqrt{Z_{open} \times Z_{short}}$$

- Geometric mean of two complex numbers***
- Calculation is trivial in polar form on Smith Chart***

Half wavelength, $l = \lambda/2$

$$Z_i = Z_L$$

Quarter wavelength, $l = \lambda/4$

$$Z_i = \frac{Z_o^2}{Z_L}$$

Eighth wavelength, $l = \lambda/8$

$$|Z_i| = Z_o \quad \text{if } Z_L \text{ and } Z_o \text{ are real (resistive)}$$

Standing Wave Ratio



$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$|\Gamma| = \frac{s - 1}{s + 1}$$

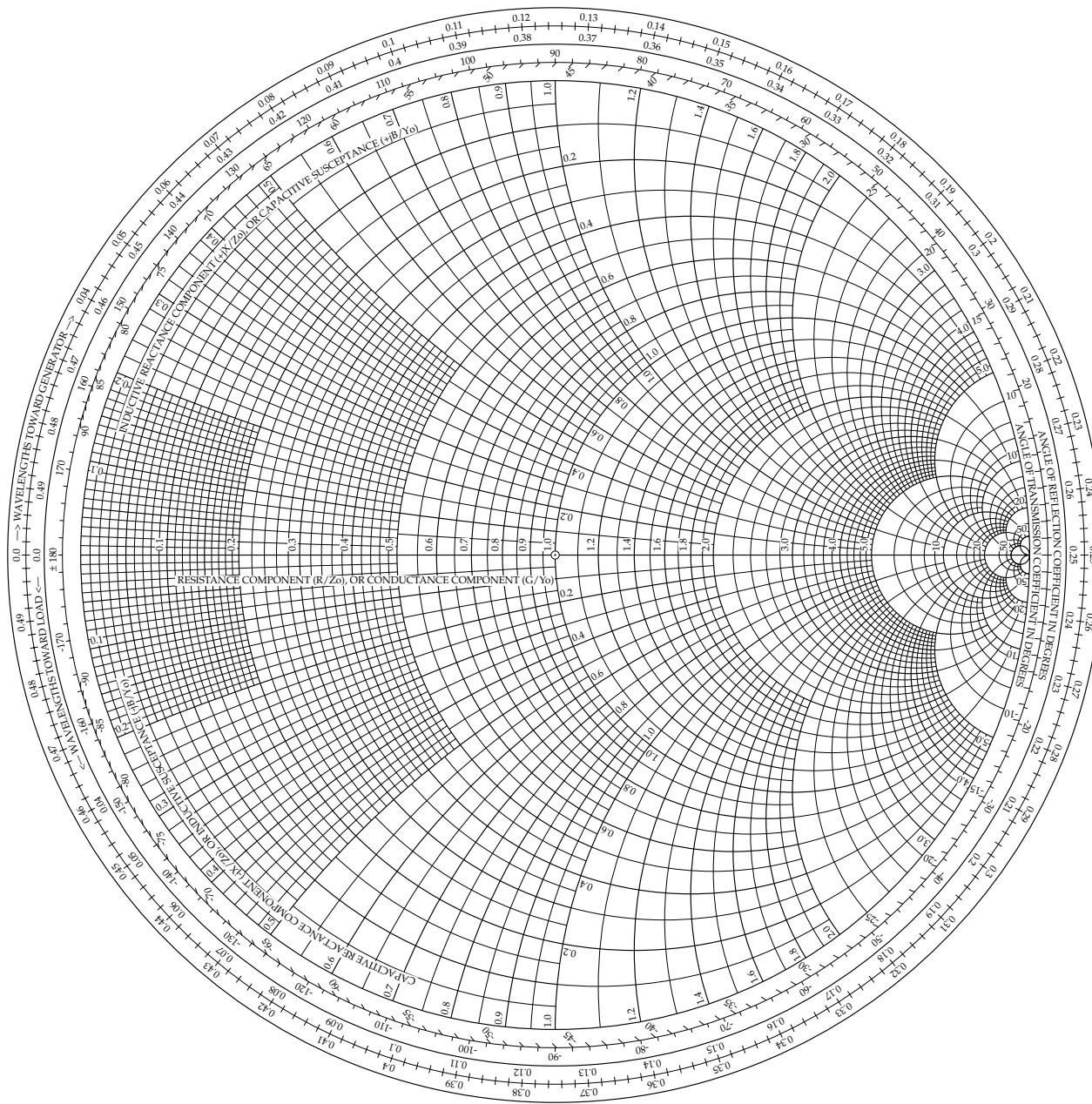
Easy to remember from

$$z = \frac{1 + \Gamma}{1 - \Gamma}$$

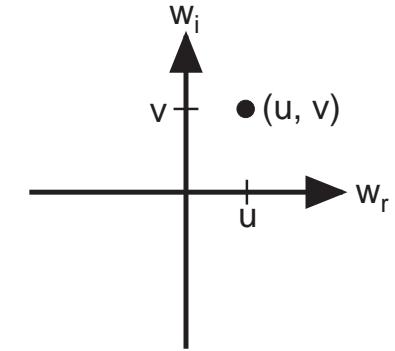
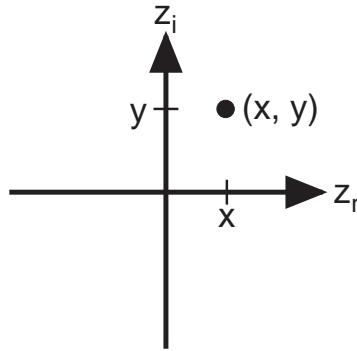
$$\Gamma = \frac{z - 1}{z + 1}$$



Part 2: The Smith Chart

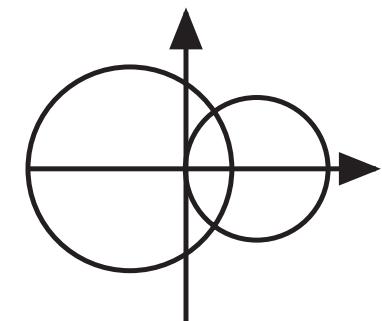
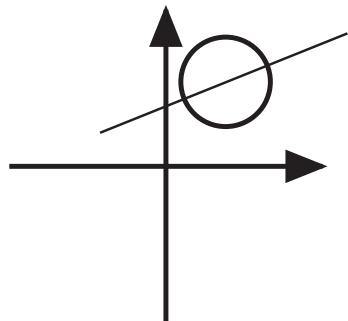


Complex Functions



□ Basic types of complex functions

- Global Properties
 - Linear – lines map to lines
 - Bilinear – circles map to circles
- Local Properties
 - Conformal – right angles map to right angles



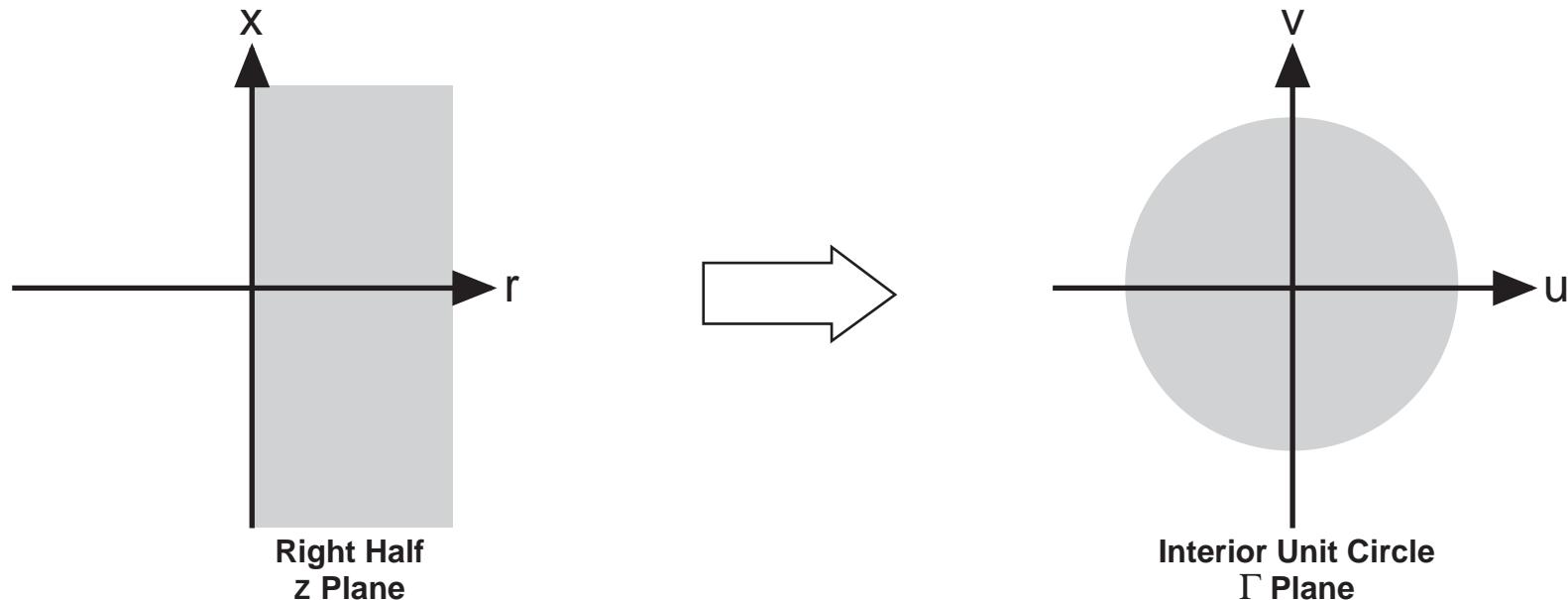
Mathematical Basis of the Smith Chart



$$\Gamma = \frac{z - 1}{z + 1}$$

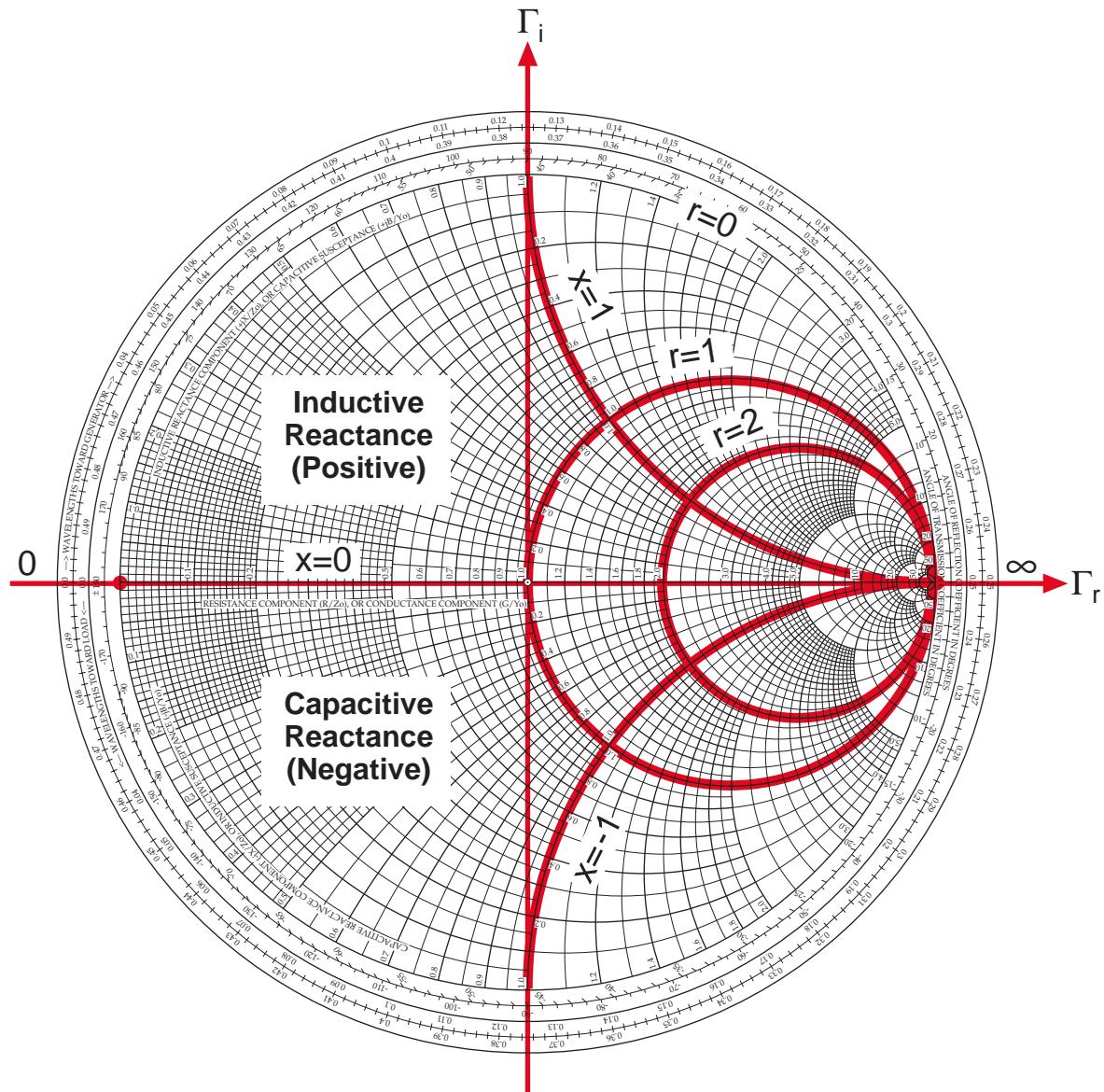
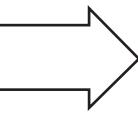
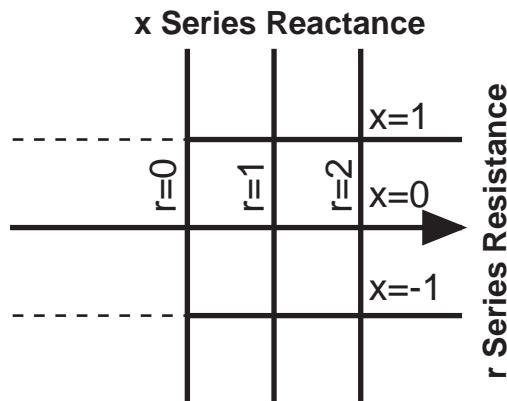
A bilinear conformal complex function

$$u + jv = \frac{(r - 1) + jx}{(r + 1) + jx}$$



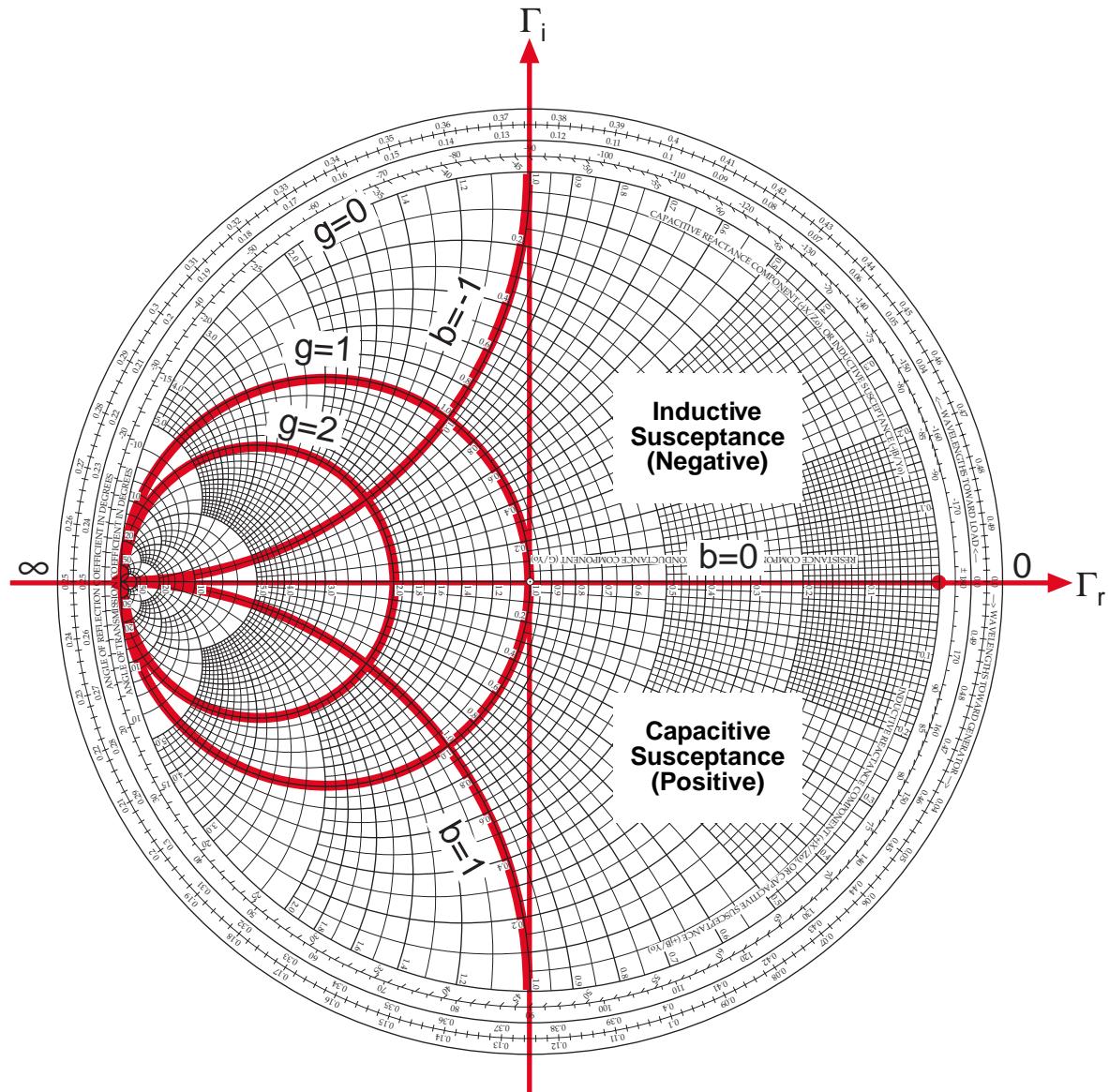
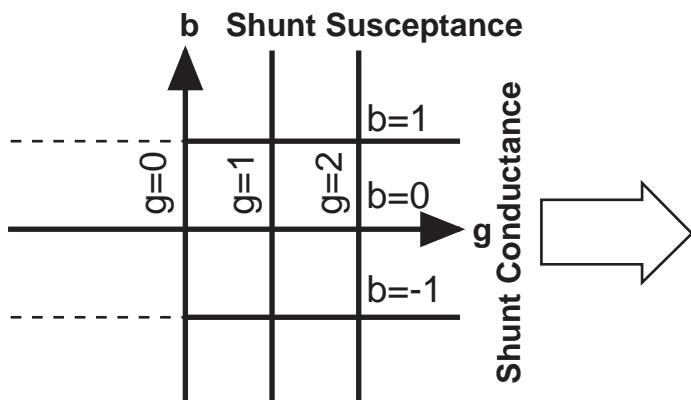
Smith Chart: Impedance Coordinates

TRW



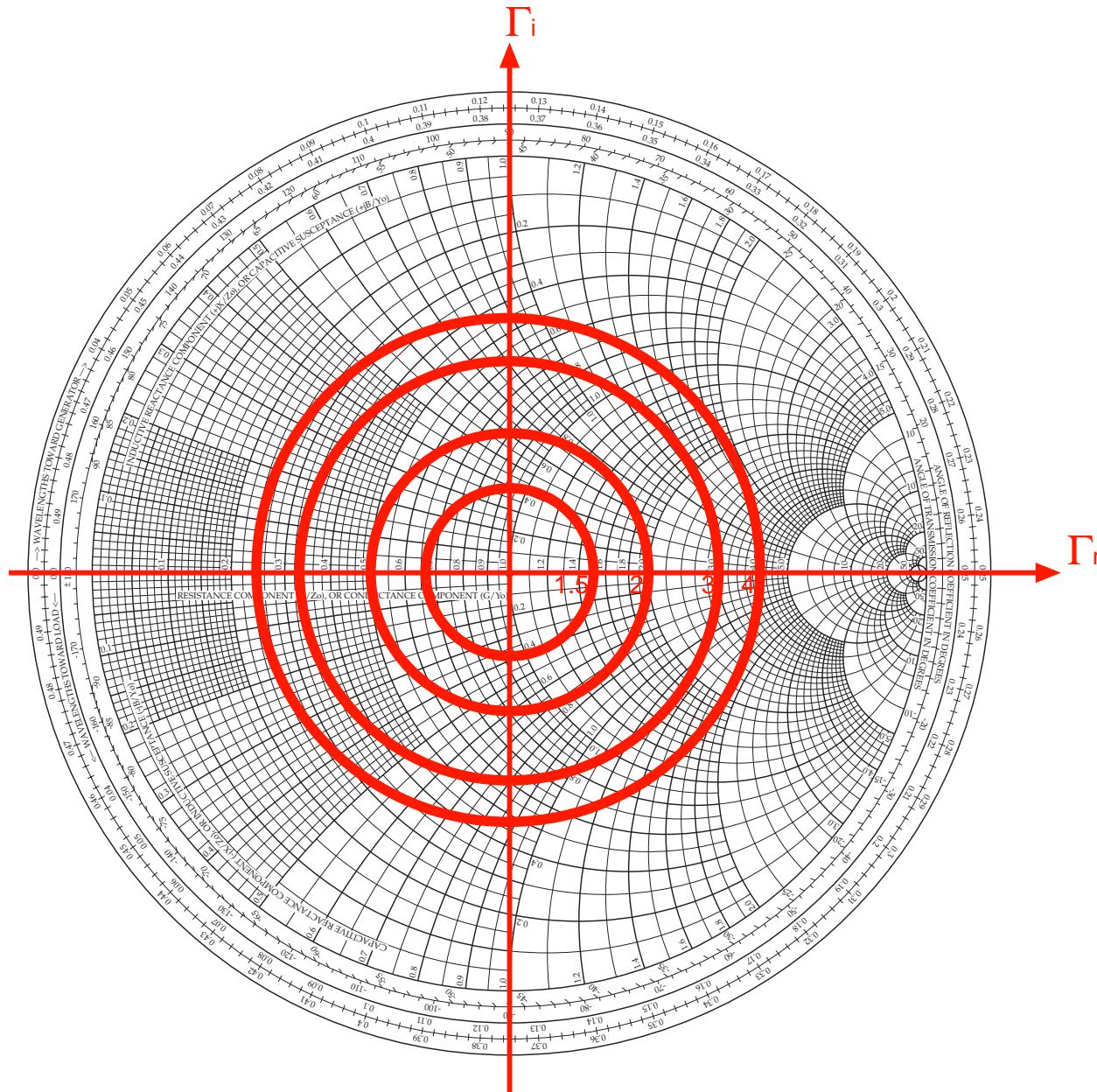
Smith Chart: Admittance Coordinates

TRW



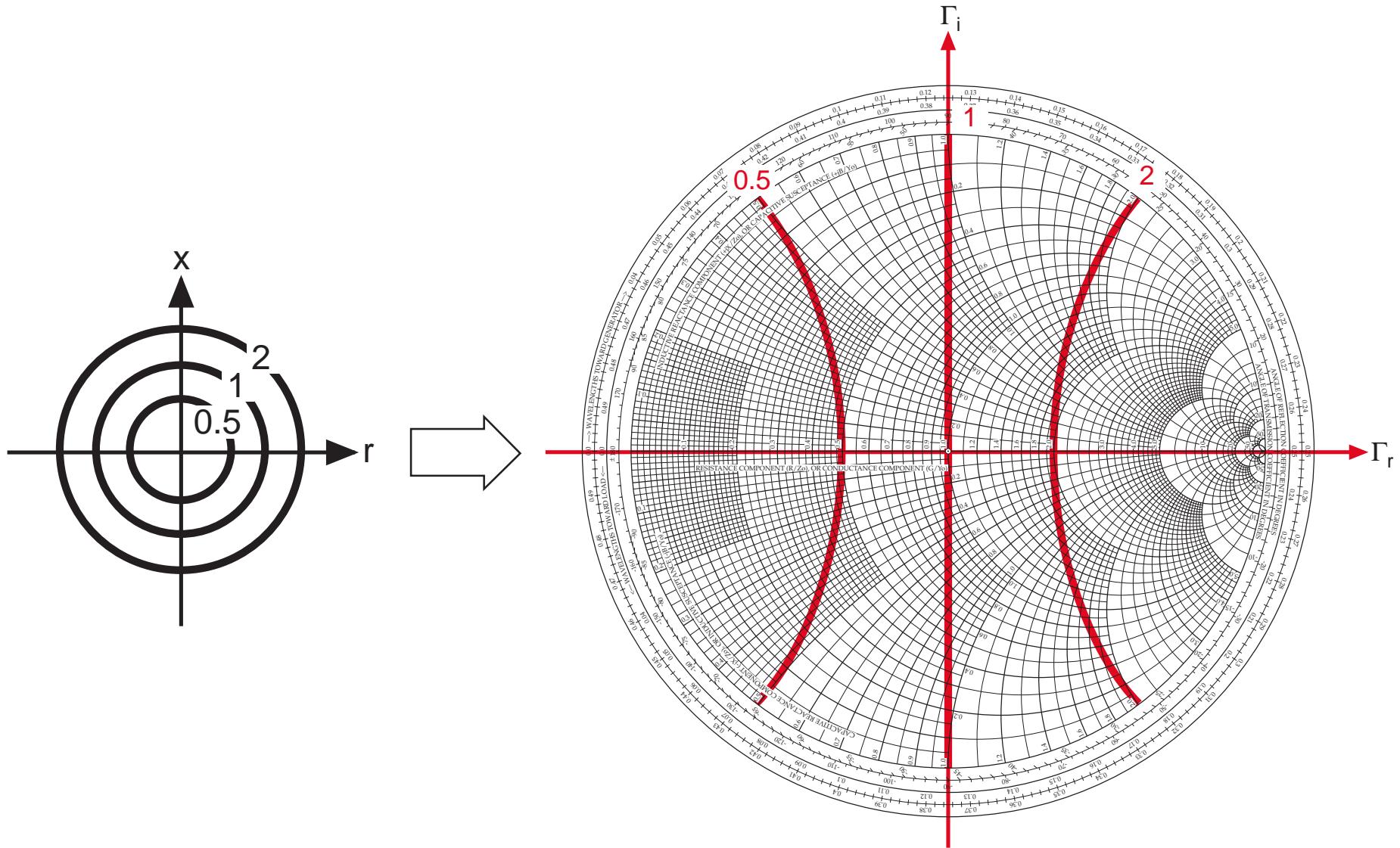
Smith Chart: Constant SWR Circles

TRW



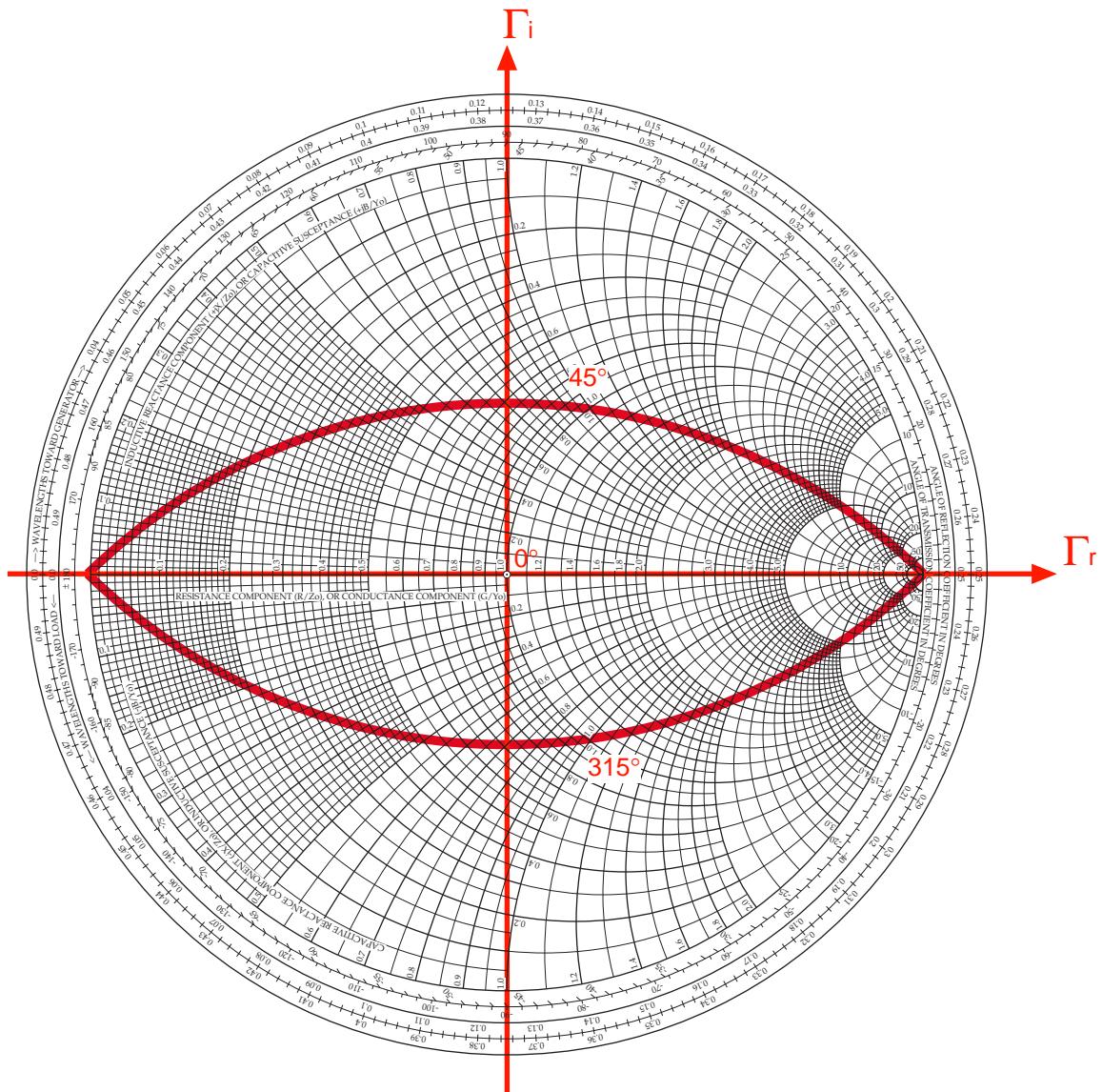
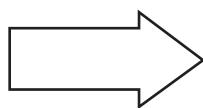
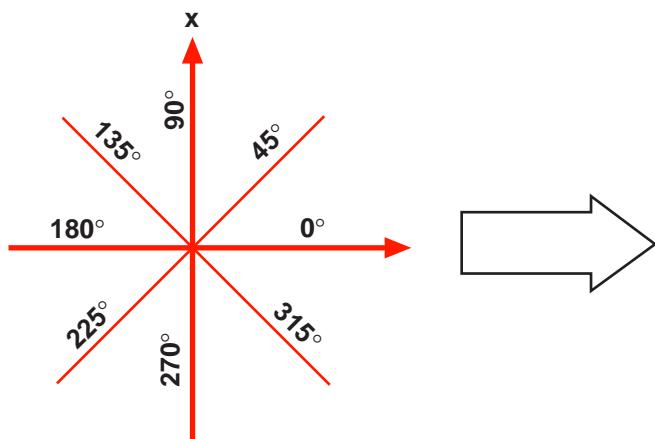
Smith Chart: Constant Impedance Magnitude Circles

TRW



Smith Chart: Constant Impedance Phase Angle Circles

TRW



Smith Chart: Multiplication, Division, Squares, and Square Roots



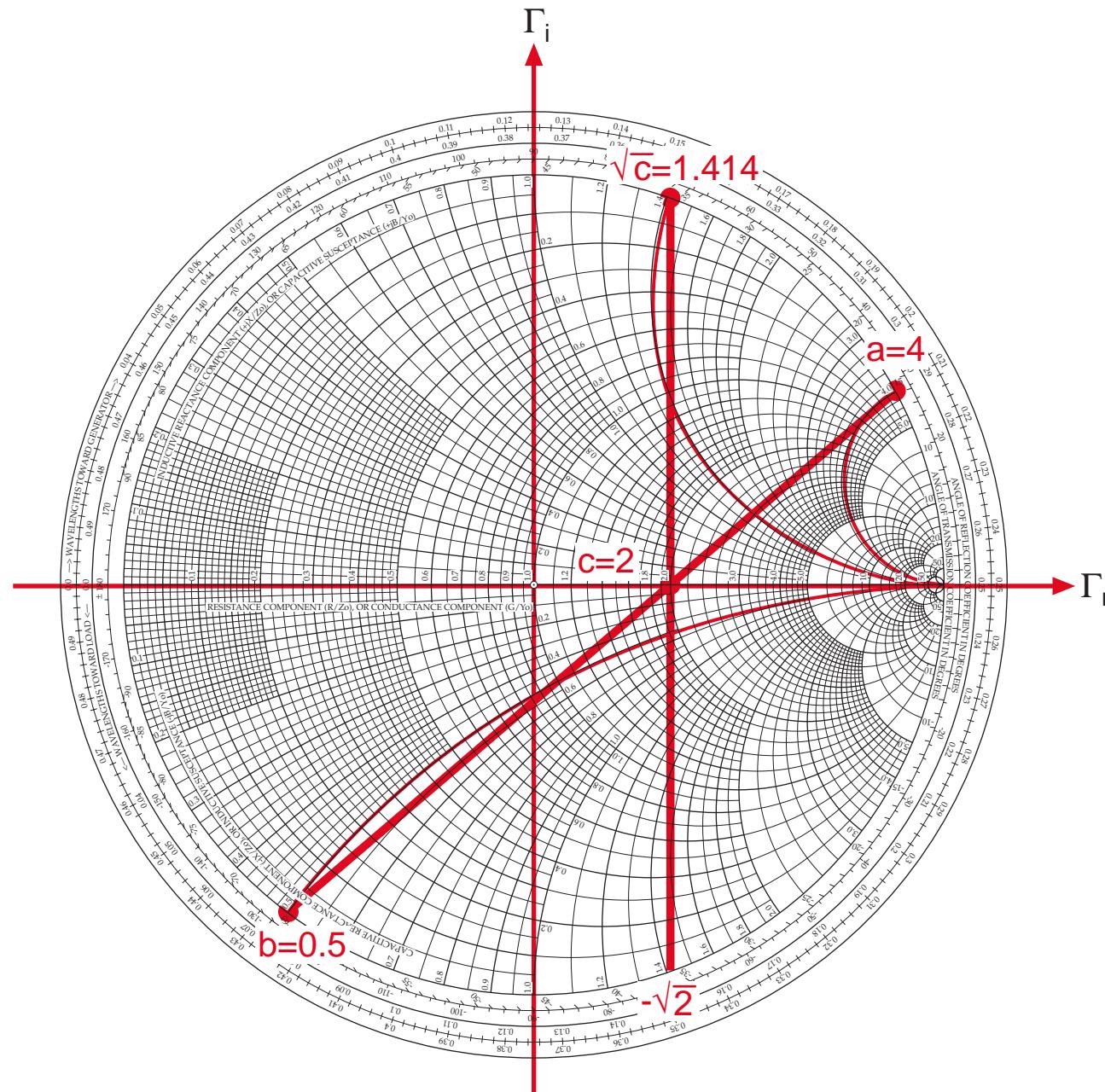
Unary Operators

<i>squares</i>	a^2
<i>square roots</i>	\sqrt{a}
<i>tangents</i>	$\tan \theta$
<i>cotangents</i>	$\cot \theta$
<i>inverse tangents</i>	$\tan^{-1} a$
<i>Inverse cotangents</i>	$\cot^{-1} a$

Binary Operators

<i>multiplication X</i>	$a \bullet b$
<i>division ÷</i>	c/a
<i>geometric mean</i>	\sqrt{ab}

Smith Chart: A Nomogram for Math Calculations





Part 3: Impedance Matching

□ Single frequency matching

- Manual synthesis using Smith Chart
- Eight canonical networks
- Lumped elements
- Series and parallel stubs
- Transmission line sections

□ Multiple frequency matching

- Ladder networks
- Multiple stubs
- Multiple line sections

□ Broadband matching

- Maximize SWR bandwidth
- Software for computer-aided manual design
- Software for network optimization
- Smith Chart used for visualization only

Fractional Bandwidth

Narrowband $< 10\%$

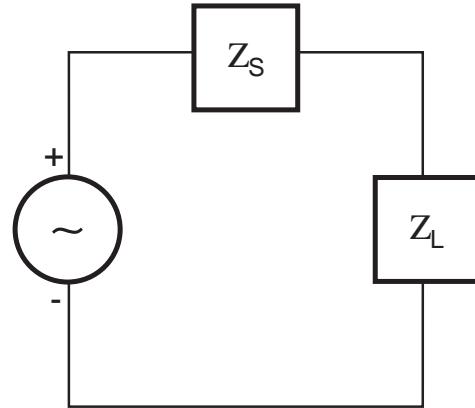
Moderate band $10\% \text{ to } 50\%$

Broadband $> 50\%$

Two Kinds of Matching



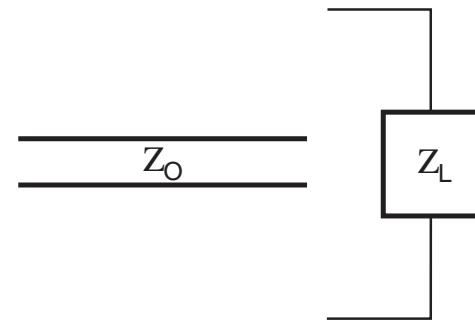
Conjugate Matching



$$Z_s = Z_L^*$$

- Best use at source (transmitter)*
- Maximizes power delivery to the load*
- Does not minimize reflections unless Z_s is real*
- Normally done by the transmitter manufacturer at the circuit design level*
- Ideally Z_s (ext) = 50 Ω*

Load Matching



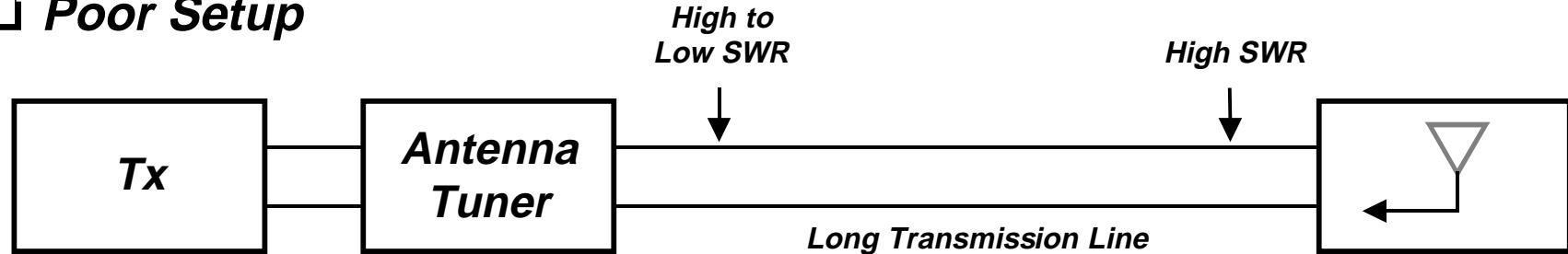
$$Z_L = Z_0$$

- Best used at ends of transmission lines*
- Minimizes reflections*
- Does not maximize delivered power unless Z_0 is real*

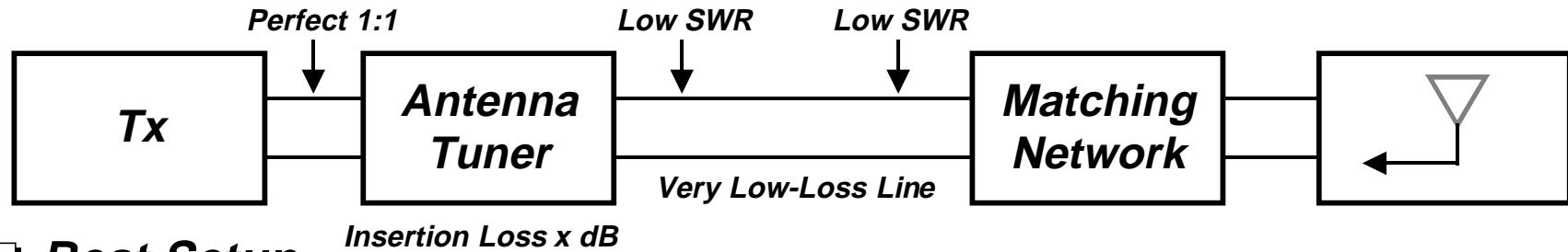
Where Should Matching Network Go?



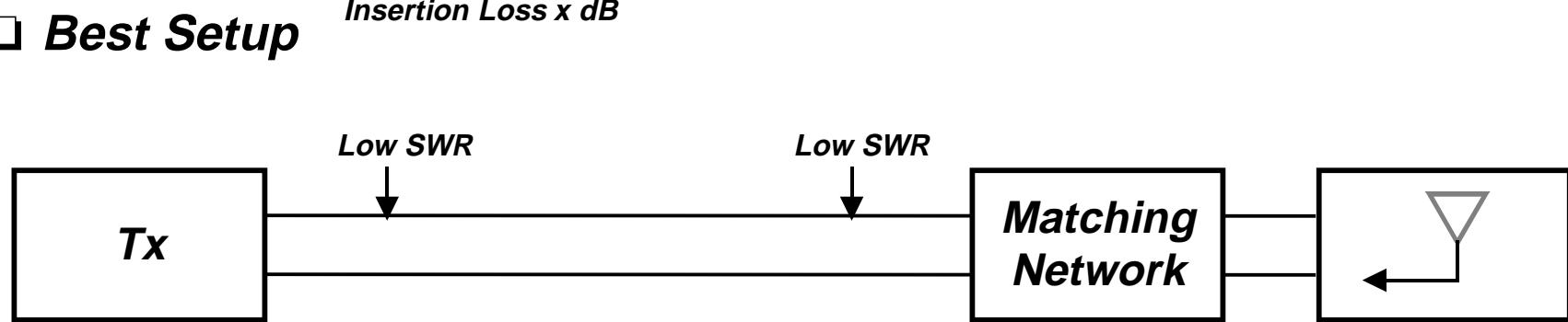
Poor Setup



Good Setup



Best Setup



- Antenna analyzer***

- Autek
 - CIA
 - MFJ

- Noise bridge (less accurate)***

- Network analyzer (more accurate)***

- Hewlett-Packard

- 1. Measure transmission line parameters Z_o , v_f**
- 2. Measure antenna feedpoint impedance across band(s) of interest**
- 3. Measure or calculate impedance across band(s) of interest at network insertion point**
- 4. Narrowband match:**
 - Select appropriate lossless L network, 2 or 4 choices
 - Select lumped elements vs stubs
 - Calculate component values
 - Calculate SWR and SWR bandwidth
 - Build and test
- 5. Broadband match:**
 - Use design software - winSMITH or equivalent
 - Design n-stage lossless ladder network
 - Select lumped elements vs stubs
 - Calculate component values
 - Calculate SWR and SWR bandwidth

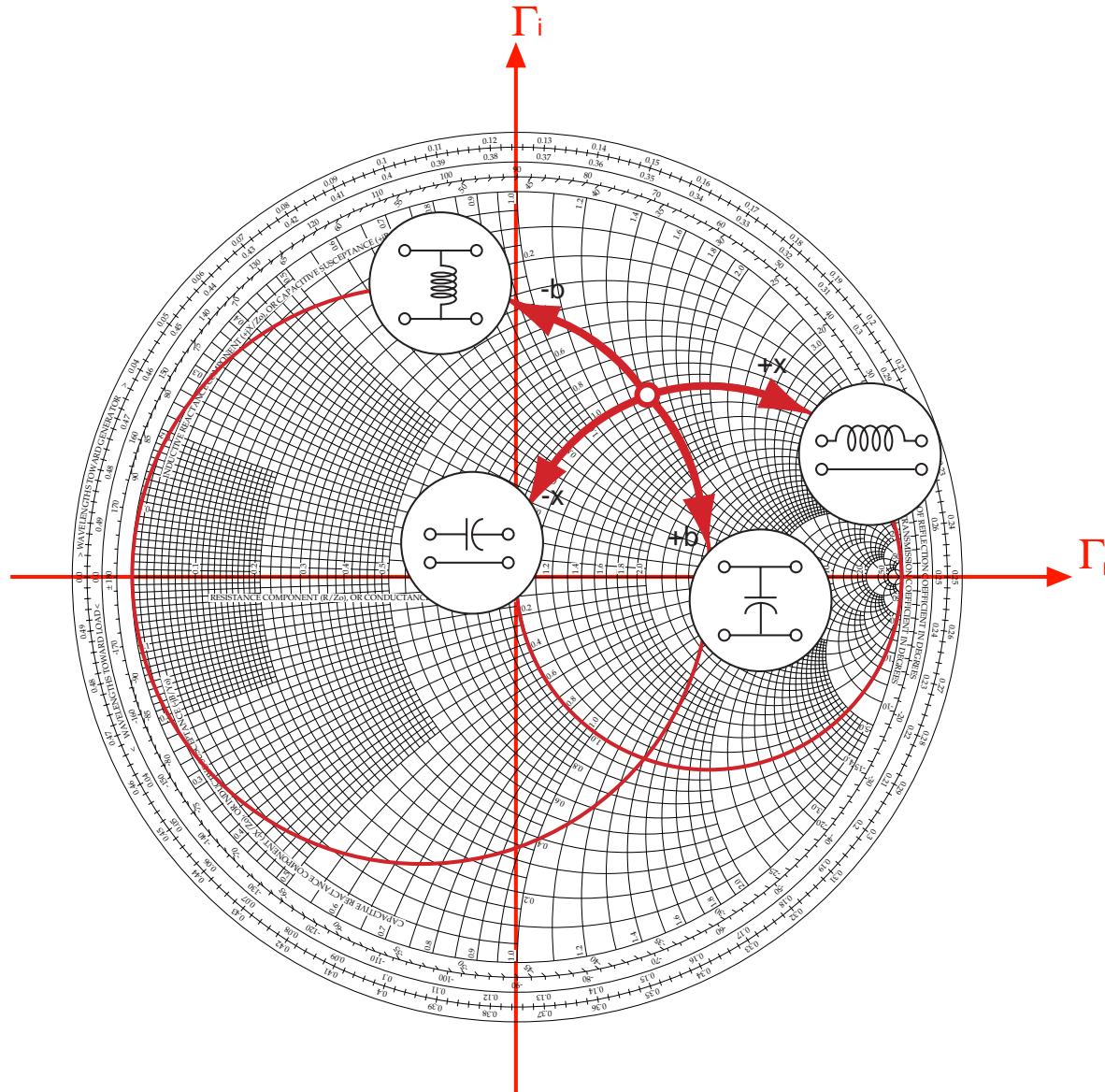
How to Measure Antenna Feedpoint Impedance

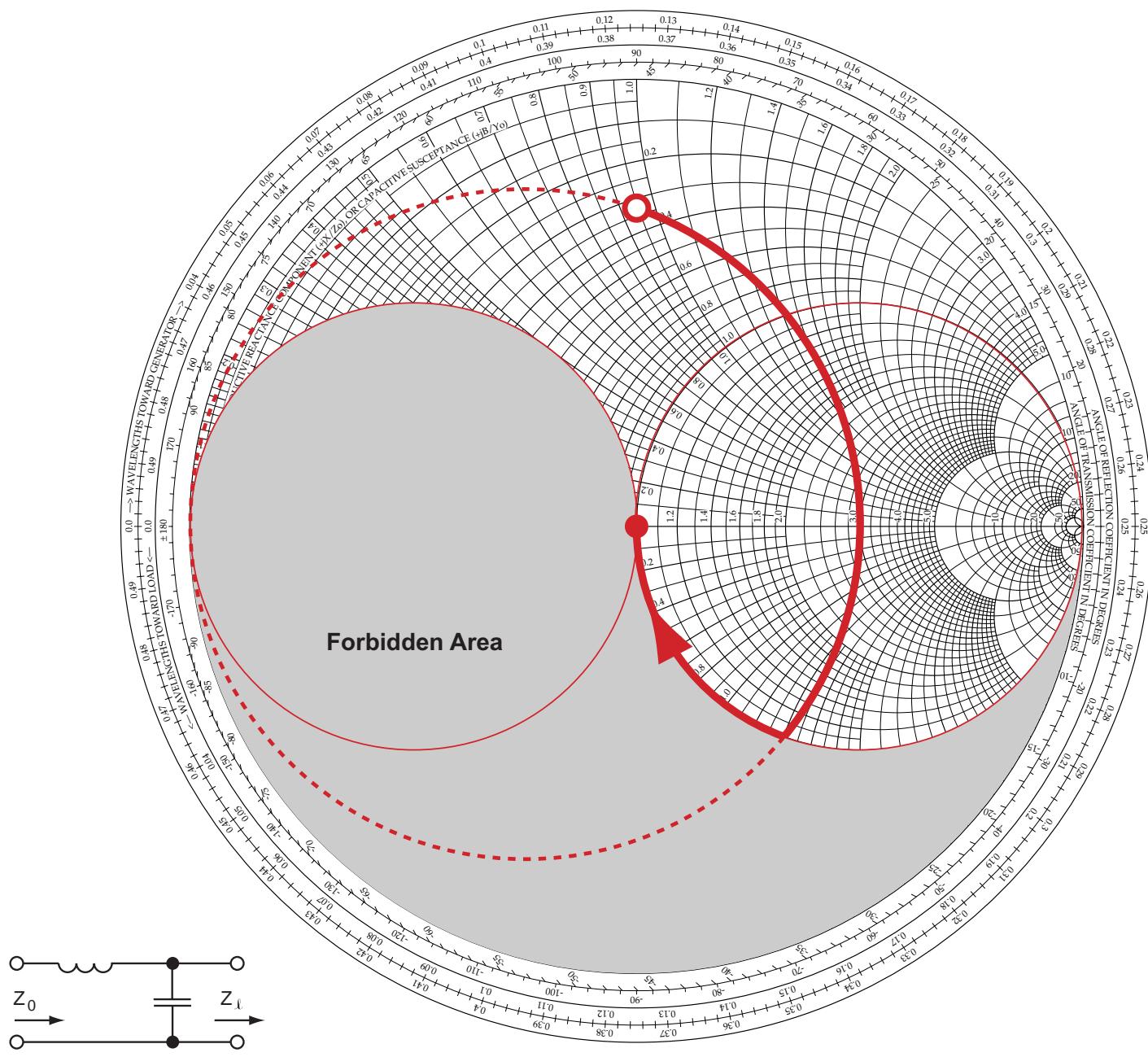


- Measure impedance through known line*
- Divide measure impedance by Z_0*
- Plot impedance point on Smith Chart*
- Move counter clockwise on chart by electrical length of line*
- Read coordinate values from chart*
- Multiply result by Z_0*

Smith Chart: Effect of Adding Series Reactances or Shunt Susceptances

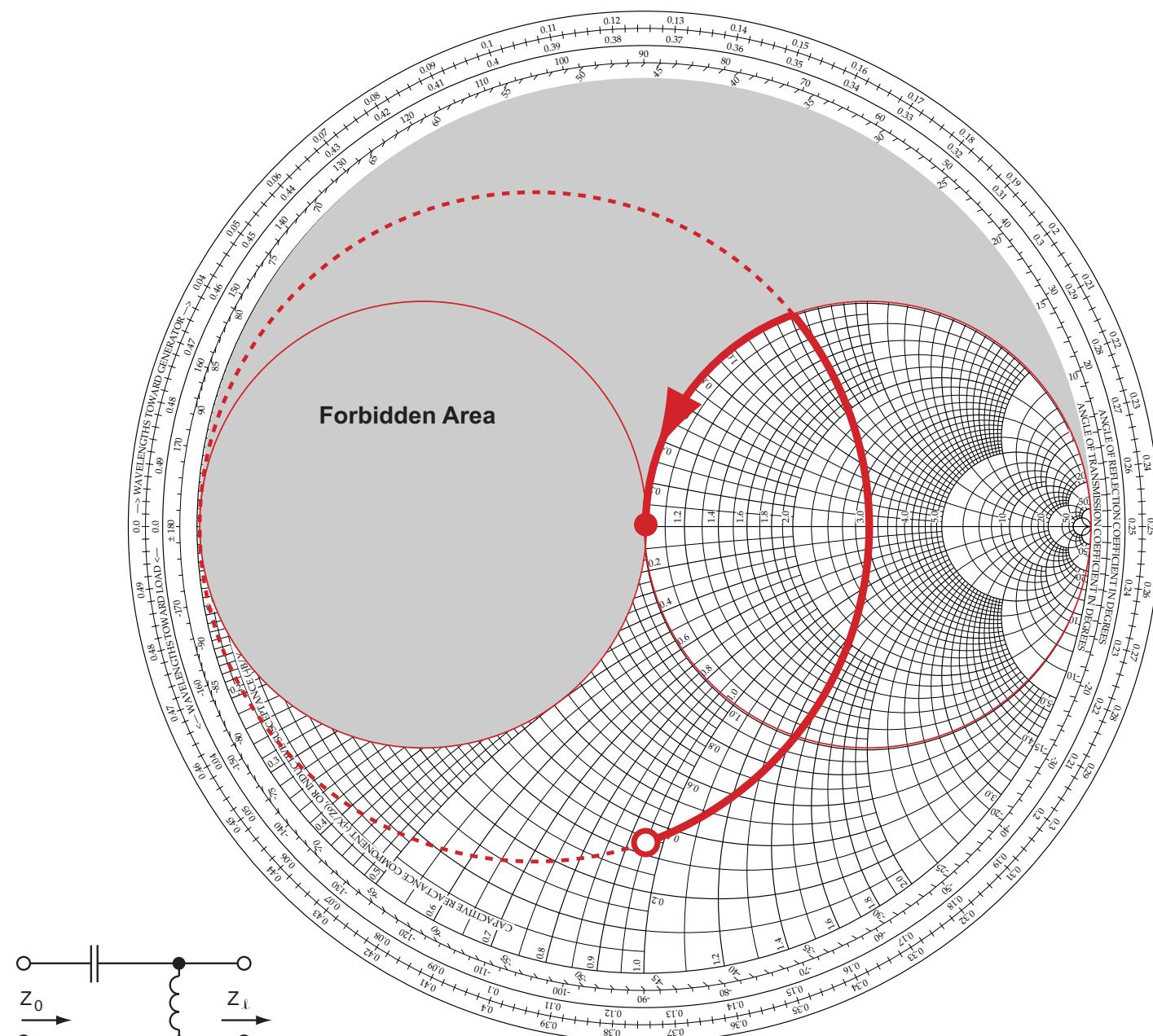
The TRW logo consists of the letters "TRW" in a bold, italicized, sans-serif font, colored orange-red.



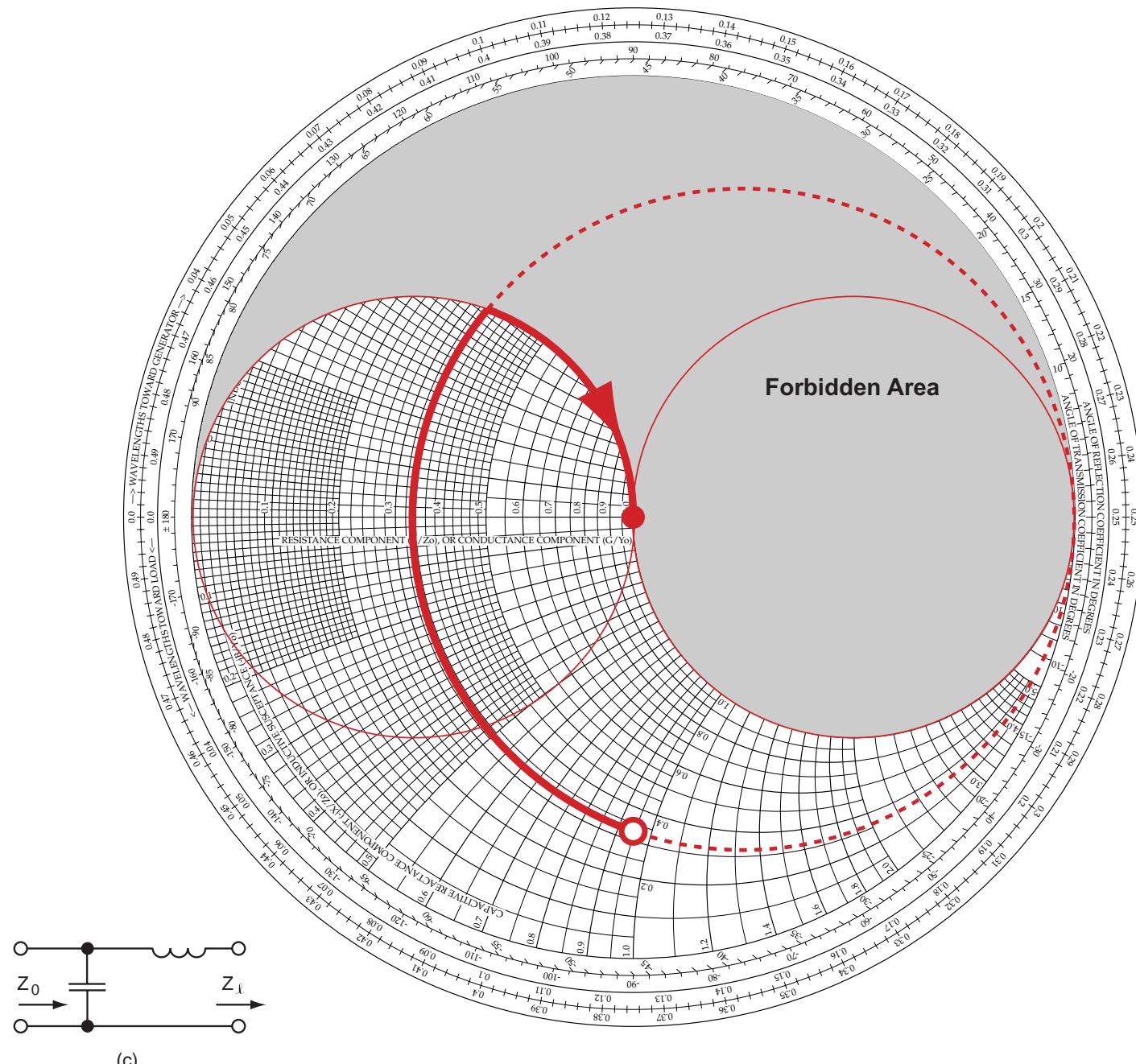


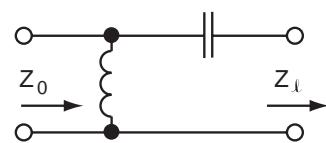
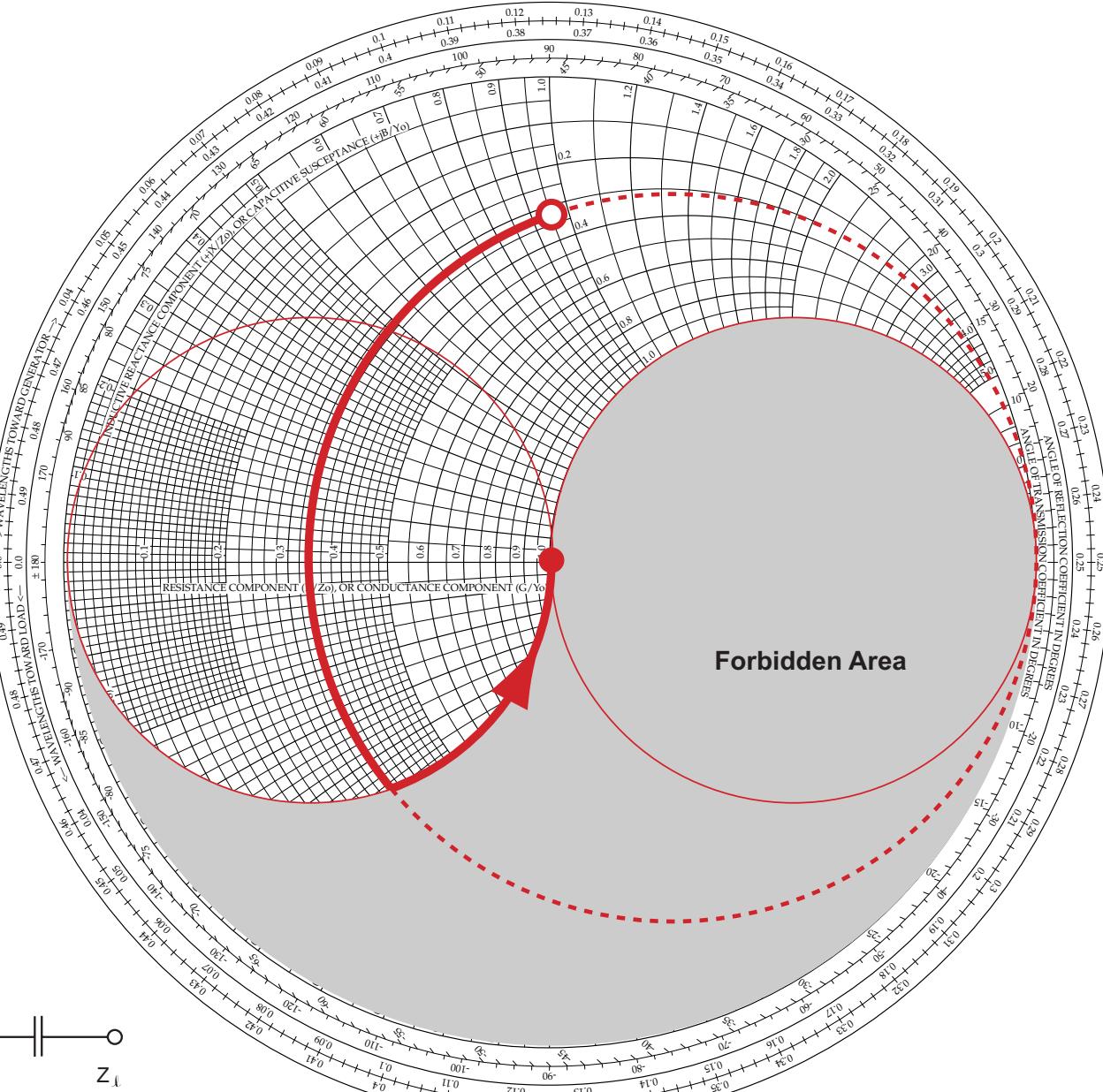
VG - 46
PSA-00527 — 10/21/2001

Pacificon 2001



(b)

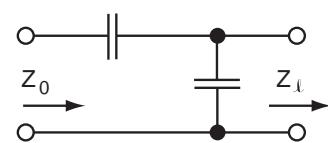
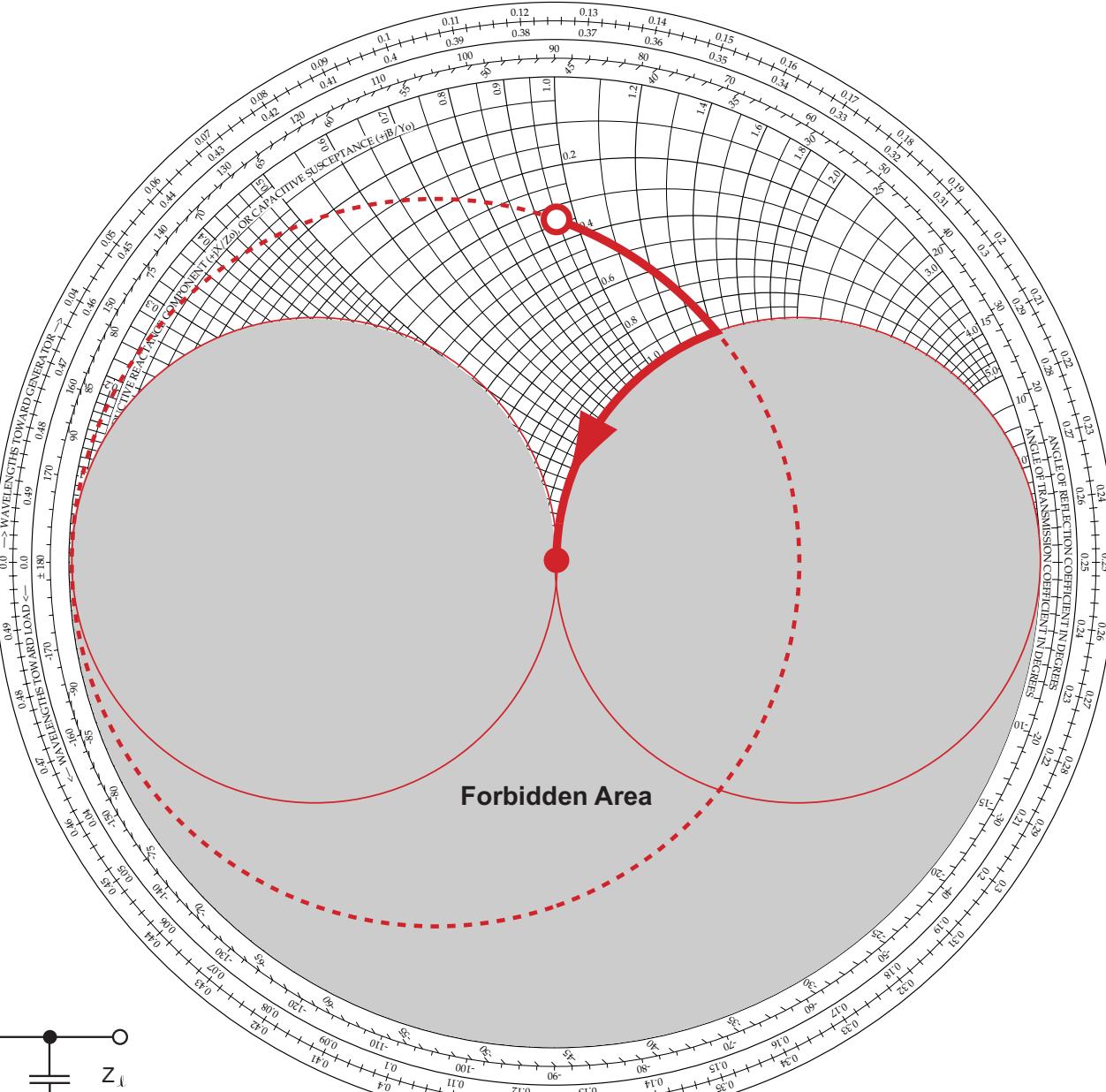




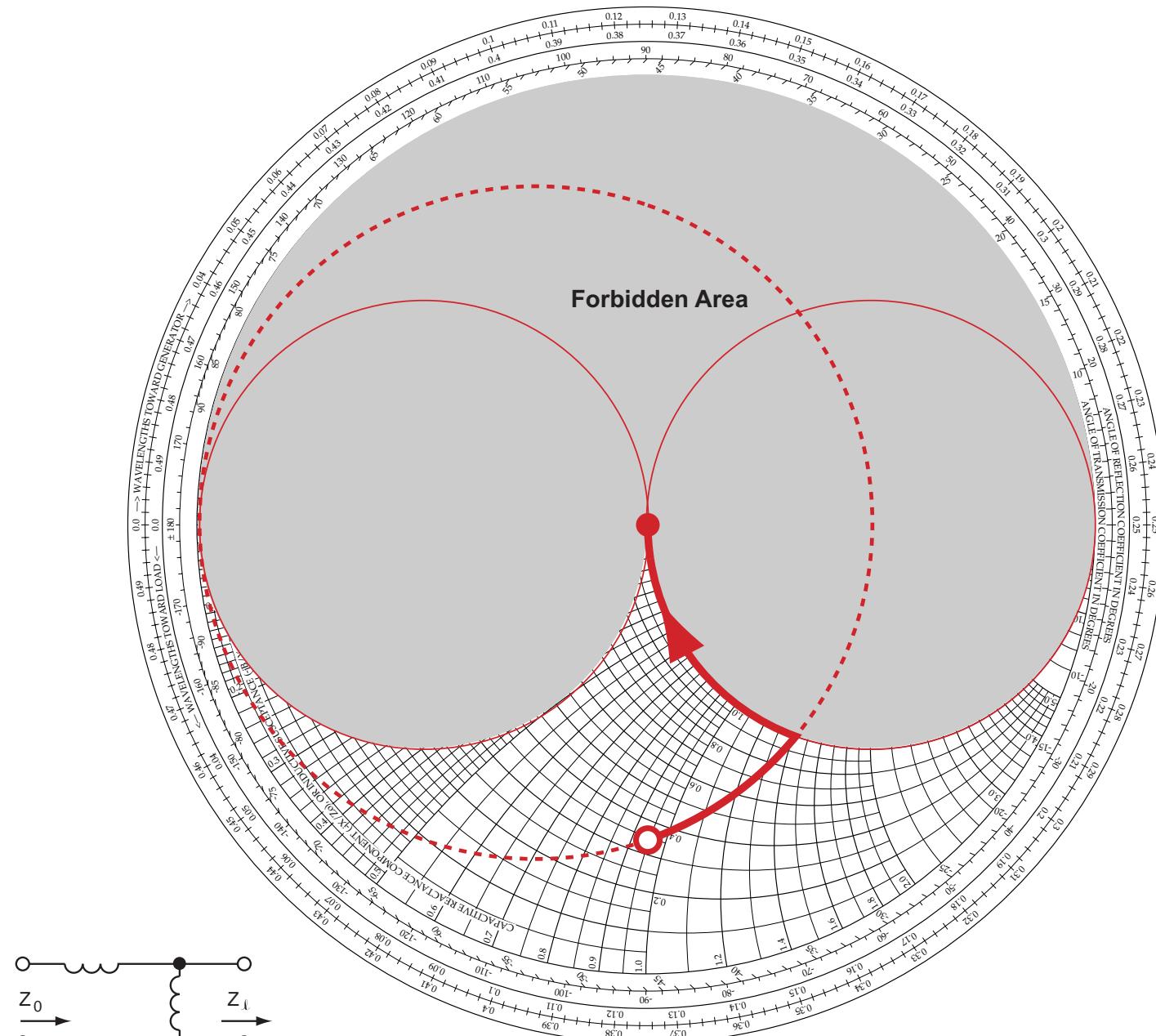
VG - 49

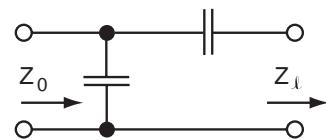
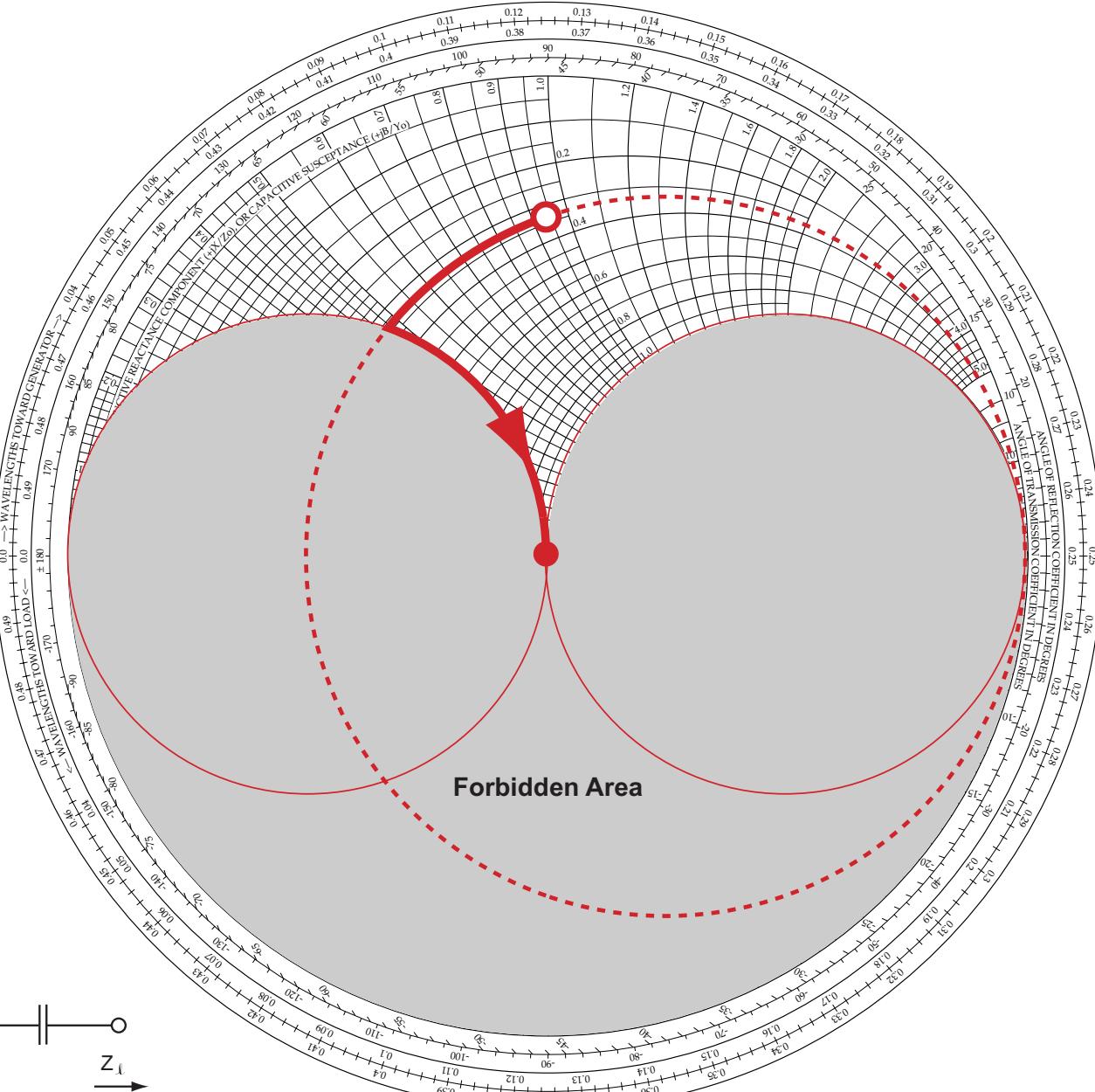
PSA-00527 — 10/21/2001

Pacificon 2001



(e)

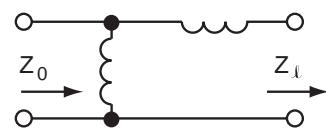
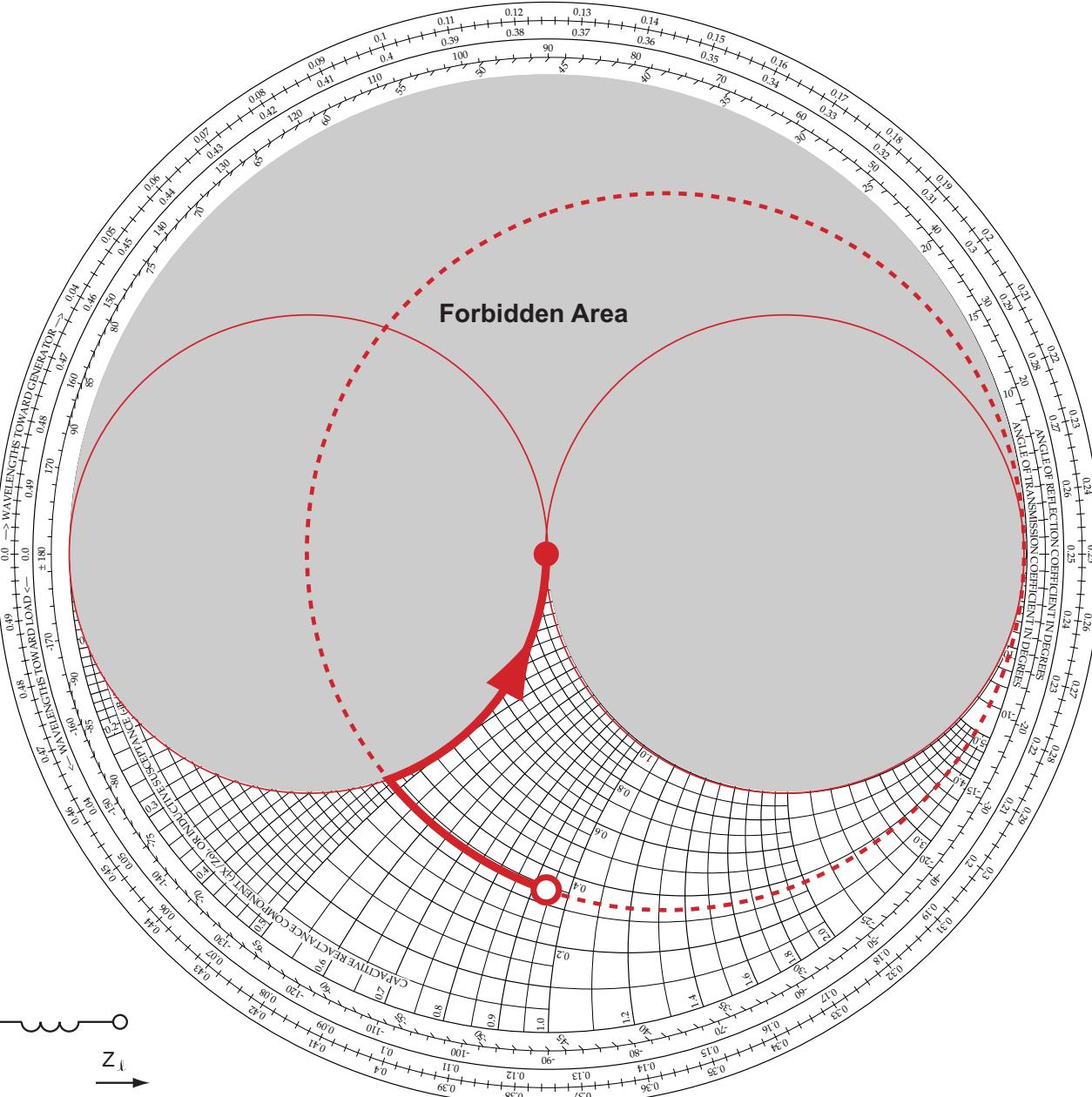




(g)

VG - 52

PSA-00527 — 10/21/2001



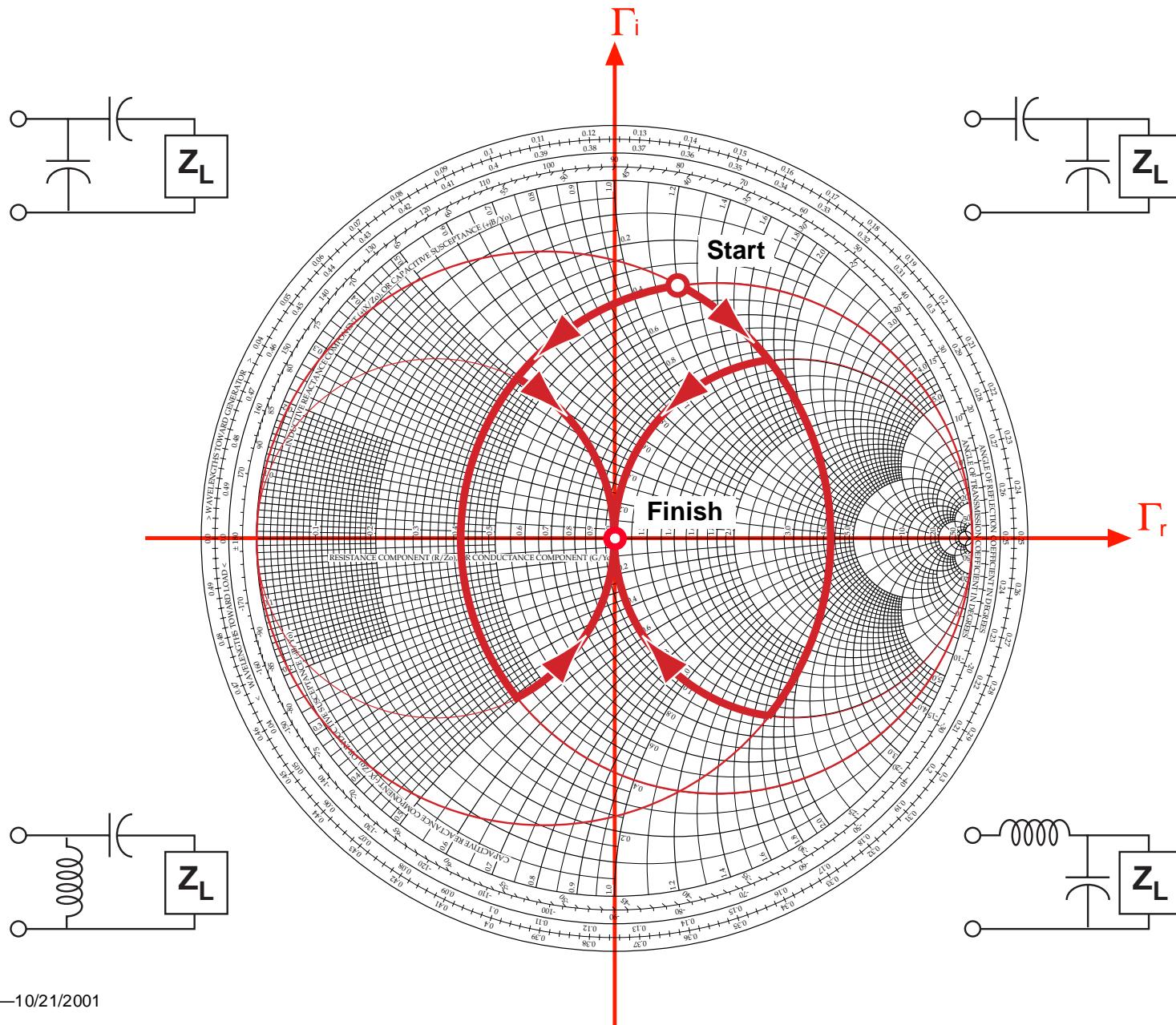
(h)

VG - 53

PSA-00527 — 10/21/2001

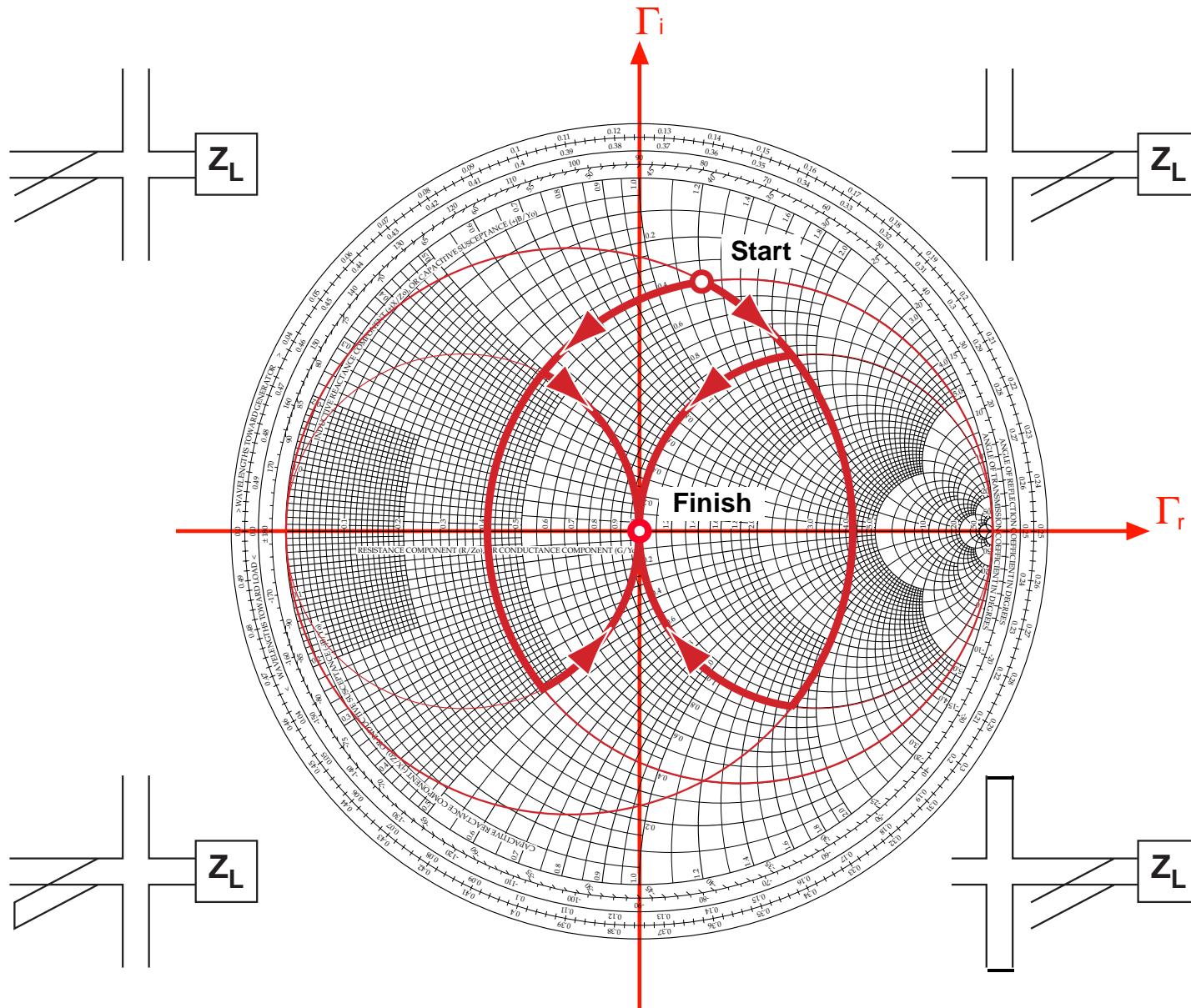
Matching: Four L-Networks Using Lumped Elements

TRW



Matching: Four L-Networks Using Stubs

TRW



Inductors

$$X_L = 2\pi fL$$

$$B_L = \frac{-1}{2\pi fL}$$

Capacitors

$$X_C = \frac{-1}{2\pi fC}$$

$$B_C = 2\pi fC$$

Assuming Z_o is real, then

Shorted Stubs _____

$$X_{shorted} = Z_o \tan\left(\frac{2\pi lf}{v_f c}\right)$$

$$B_{shorted} = \frac{-1}{Z_o \tan\left(\frac{2\pi lf}{v_f c}\right)}$$

Open Stubs _____

$$X_{open} = \frac{-Z_o}{\tan\left(\frac{2\pi lf}{v_f c}\right)}$$

$$B_{open} = \frac{1}{Z_o} \tan\left(\frac{2\pi lf}{v_f c}\right)$$

Broadband Matching Network Design Recipe

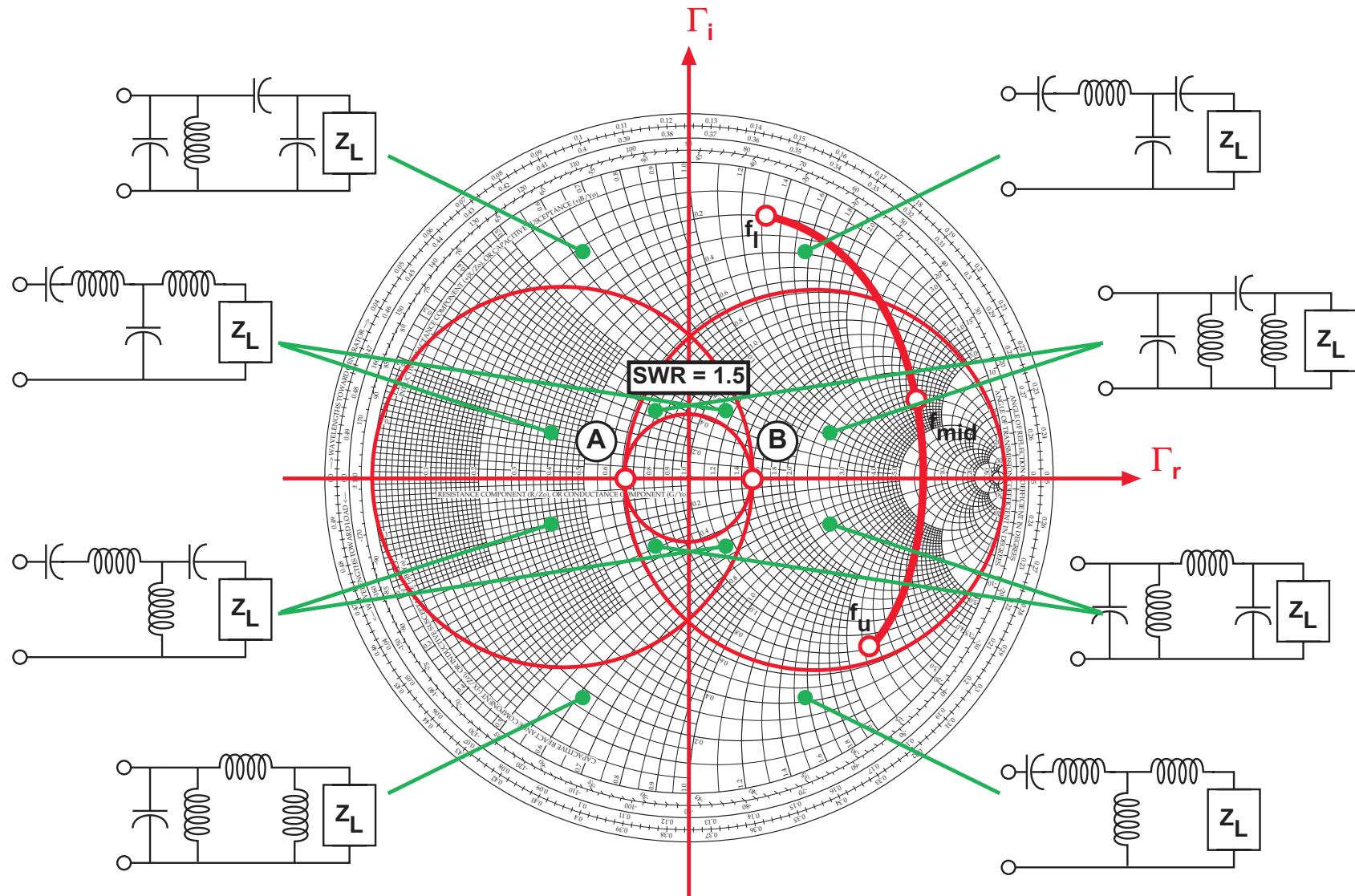
Using 4-Element π -Resonant and T-Resonant Networks



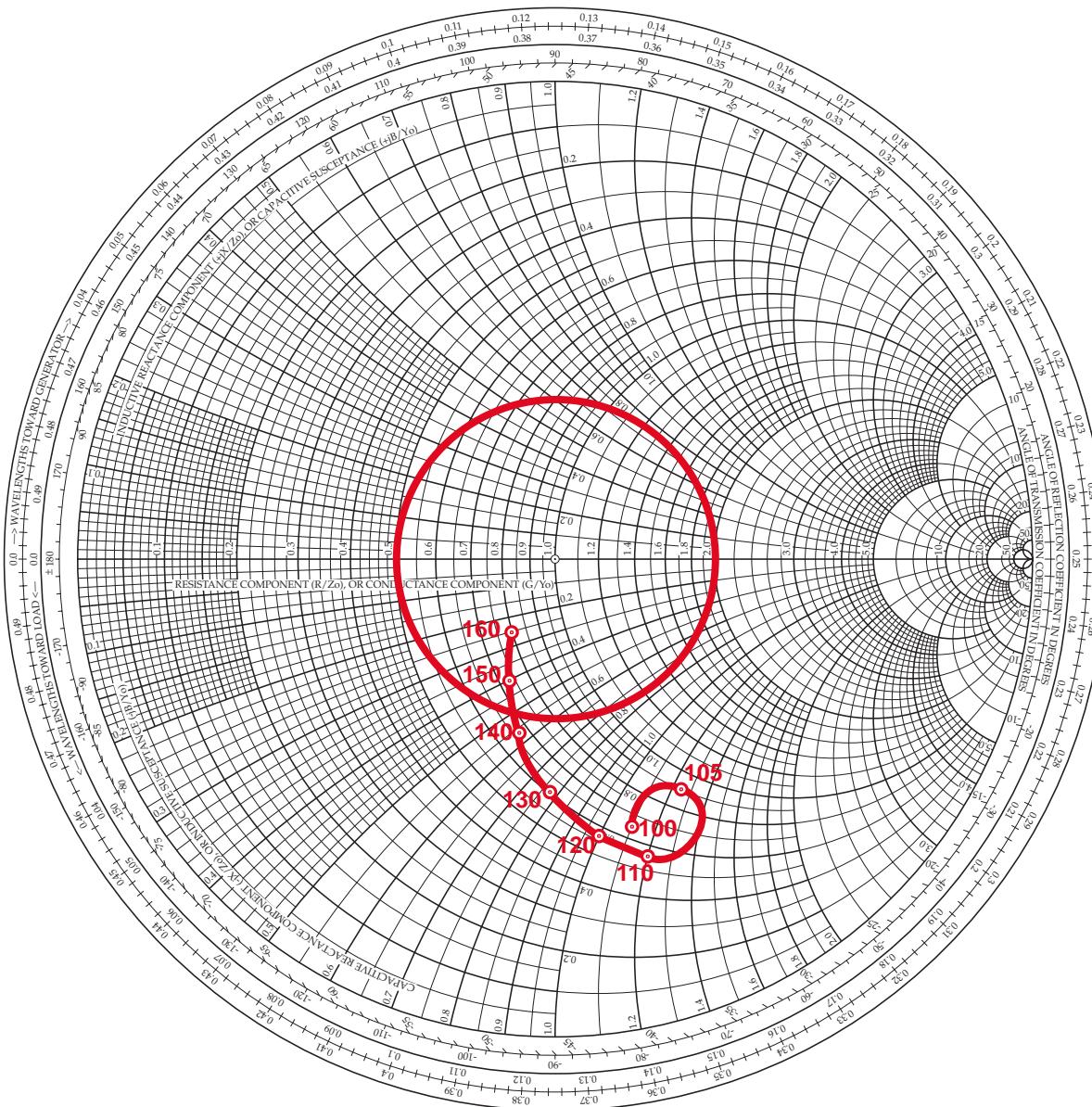
- Putting network insertion point close to load (antenna) gives greatest SWR bandwidth***
- Step 1: Using an L-network, move the midband impedance point to prime point A or B. Bandwidth will be maximized if the minimum reactance or susceptance L-network is chosen in this step.***
- Step 2: Wrap the impedance locus into the SWR circle by adding a series or parallel resonant circuit as required to complete the π -resonant or T-resonant network***

Π -Resonant and T-Resonant Network For Moderate and Broadband Matching

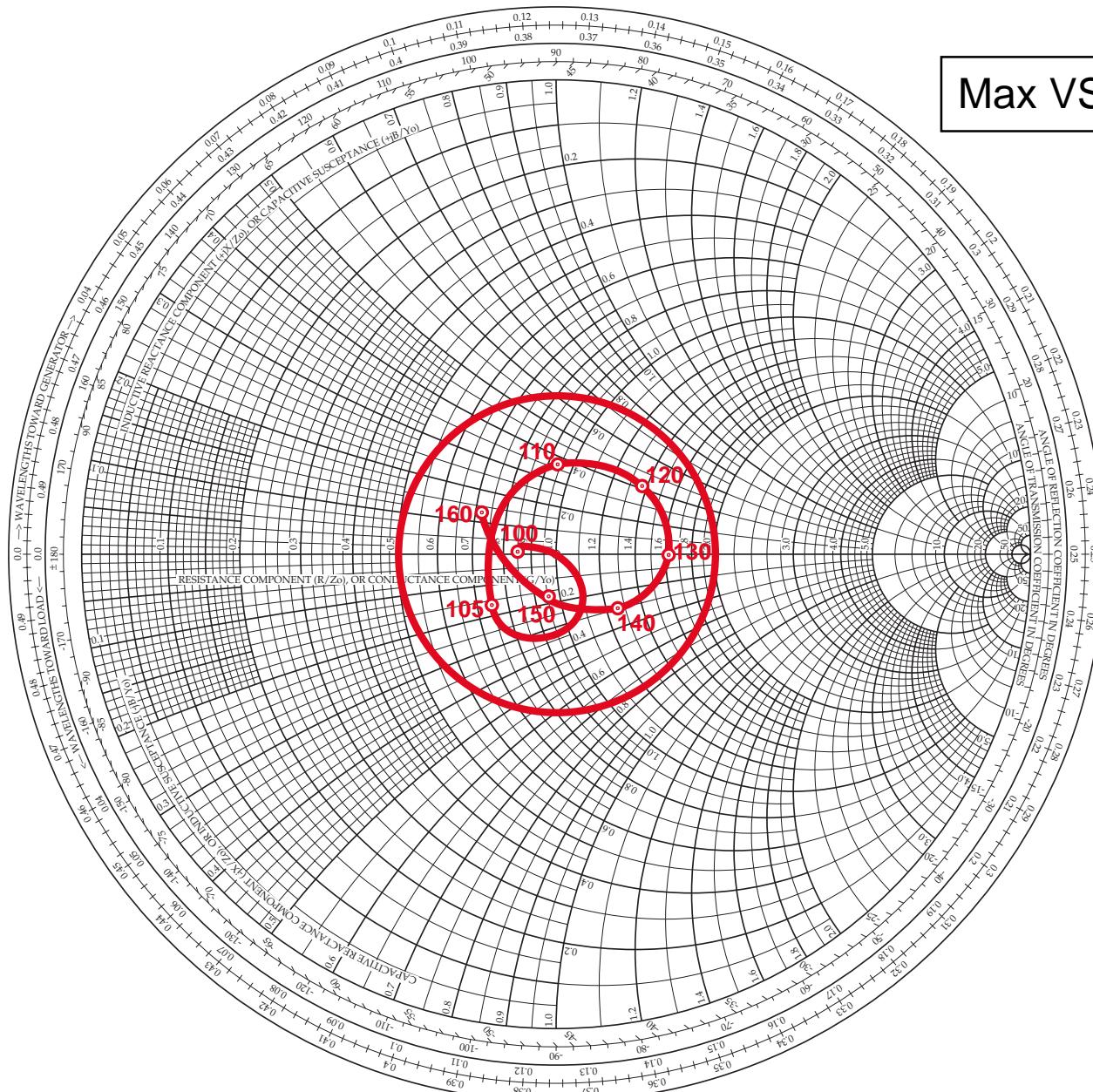
TRW



Caron's Example 10: VHF Folded Blade Antenna



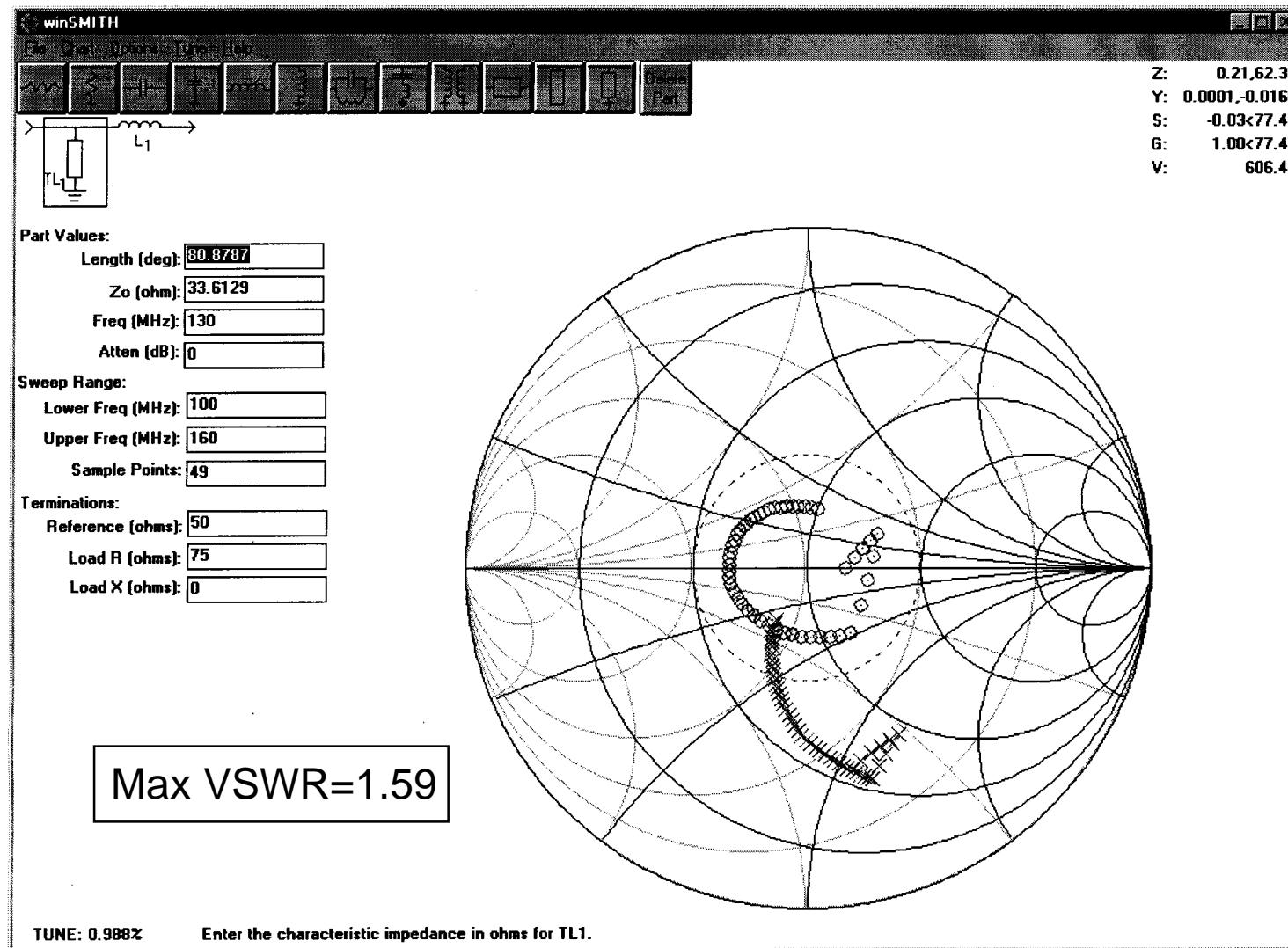
Caron's Matching Solution

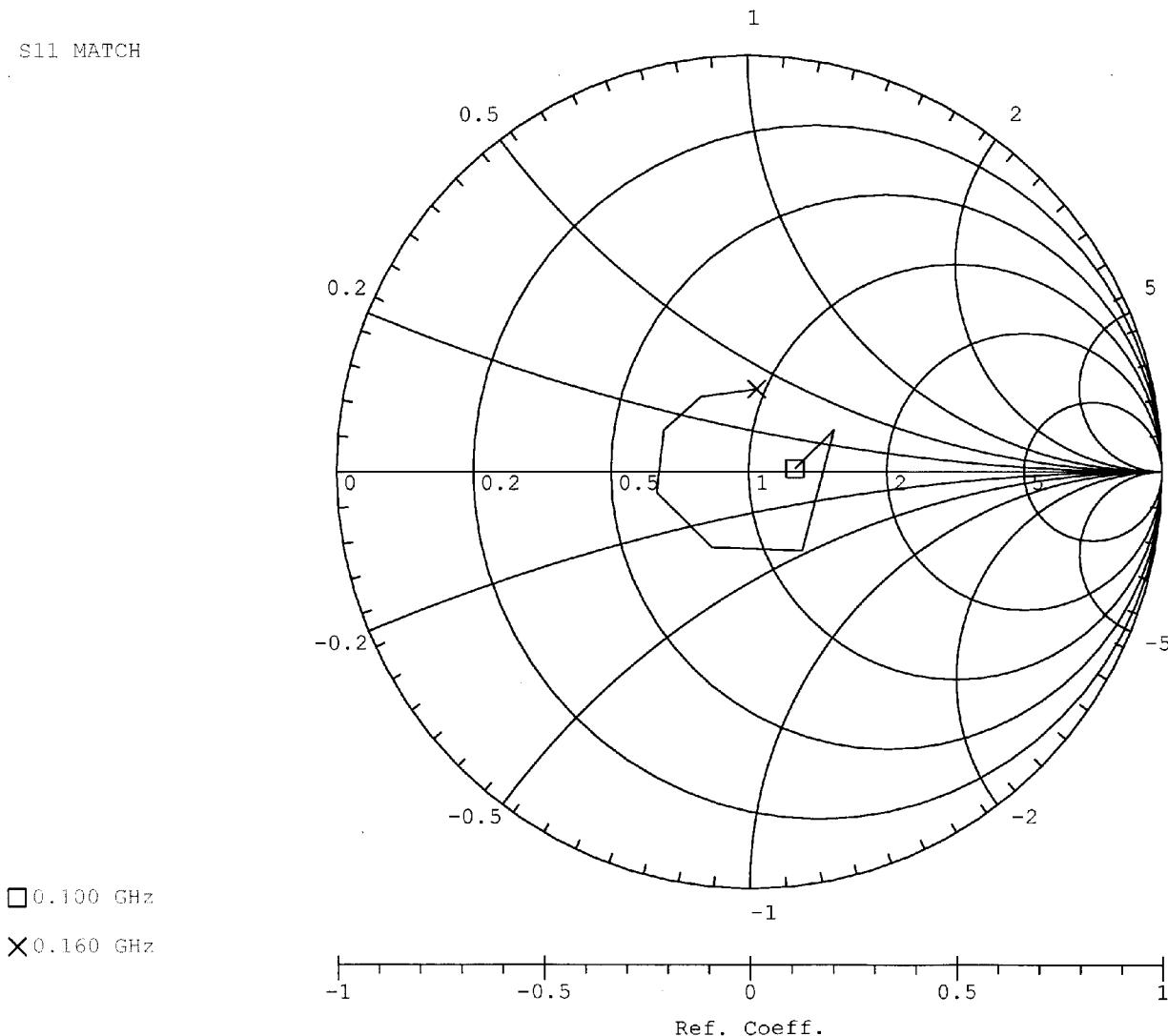


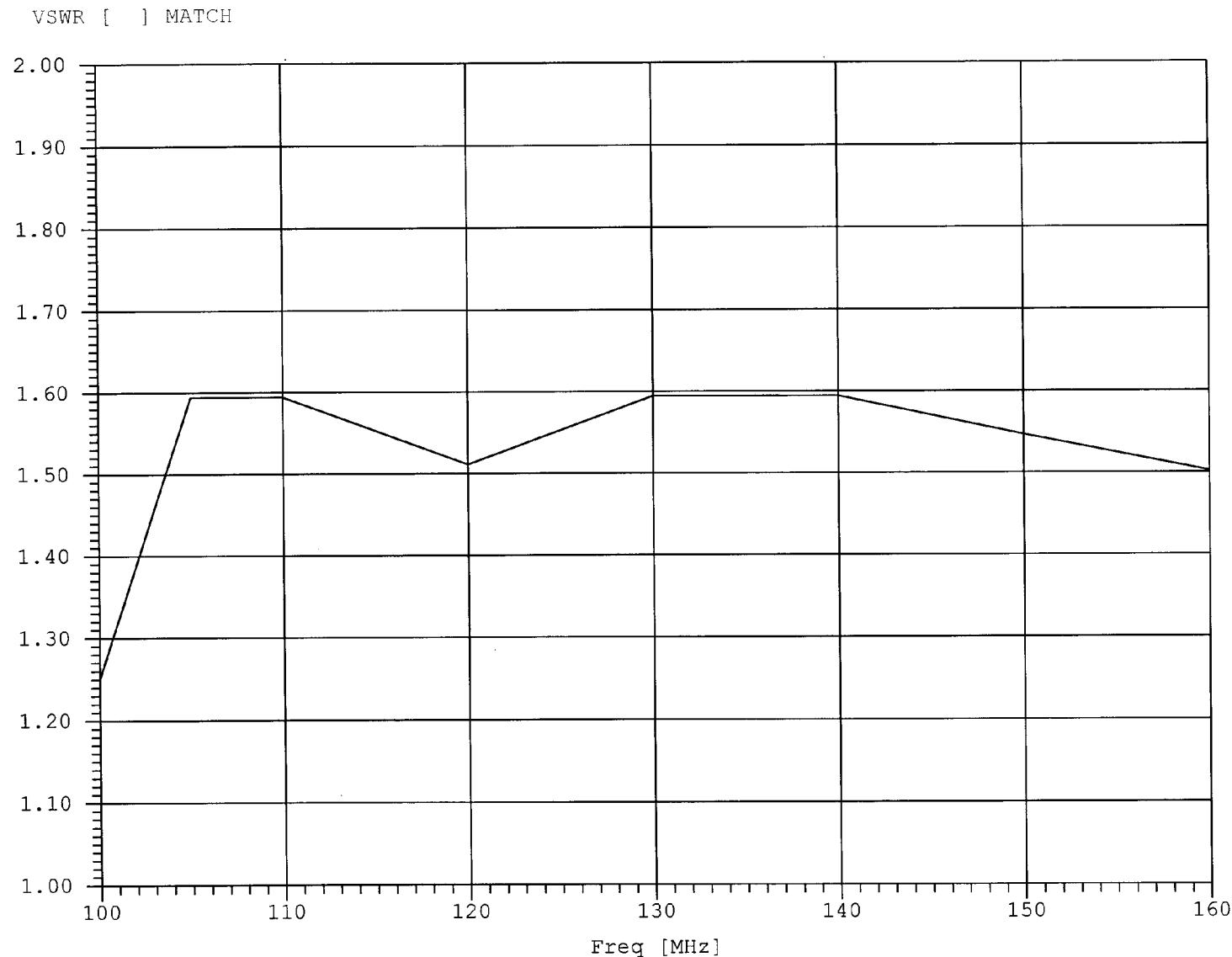
Max VSWR=1.67

WinSmith Display

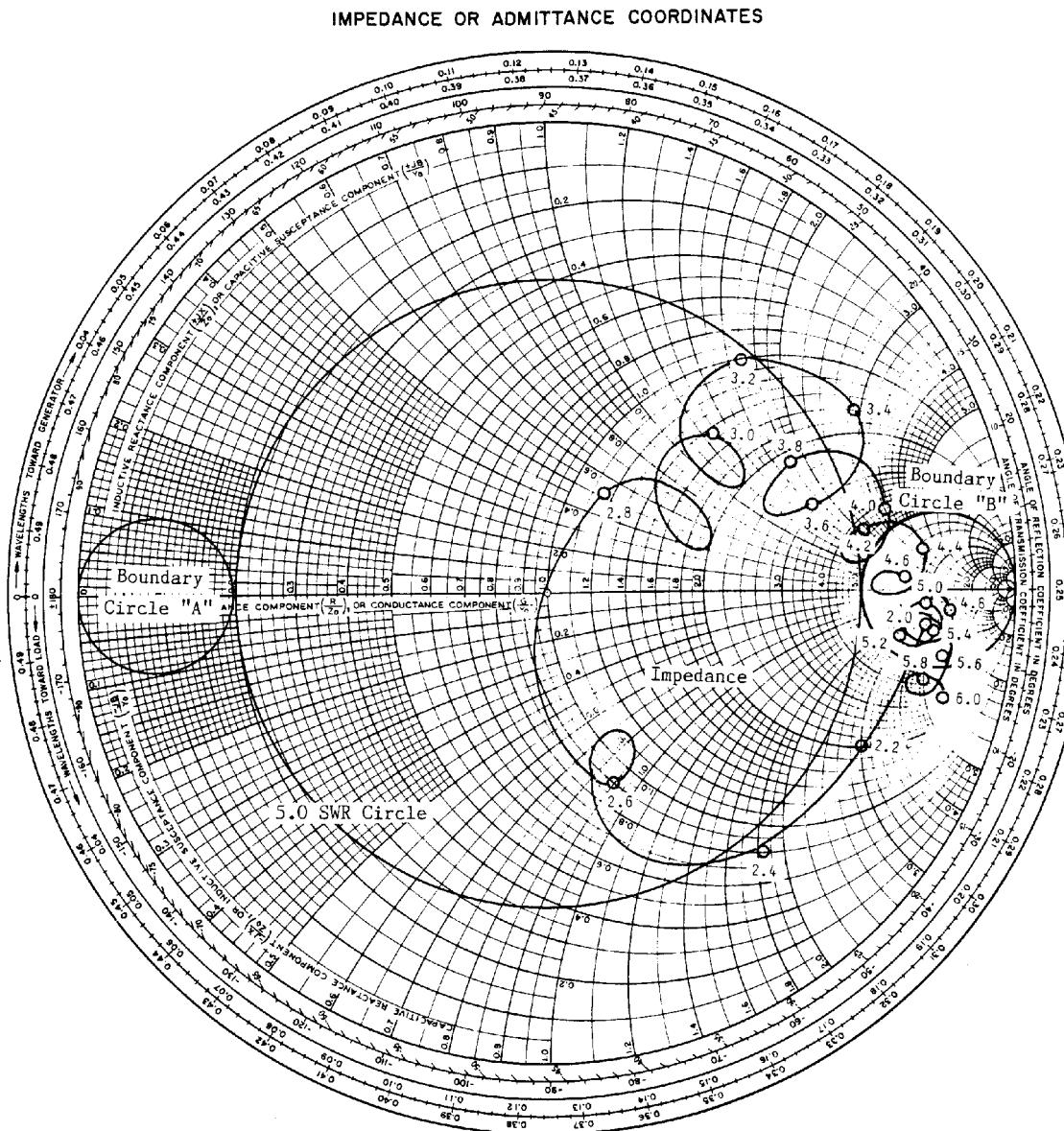
TRW





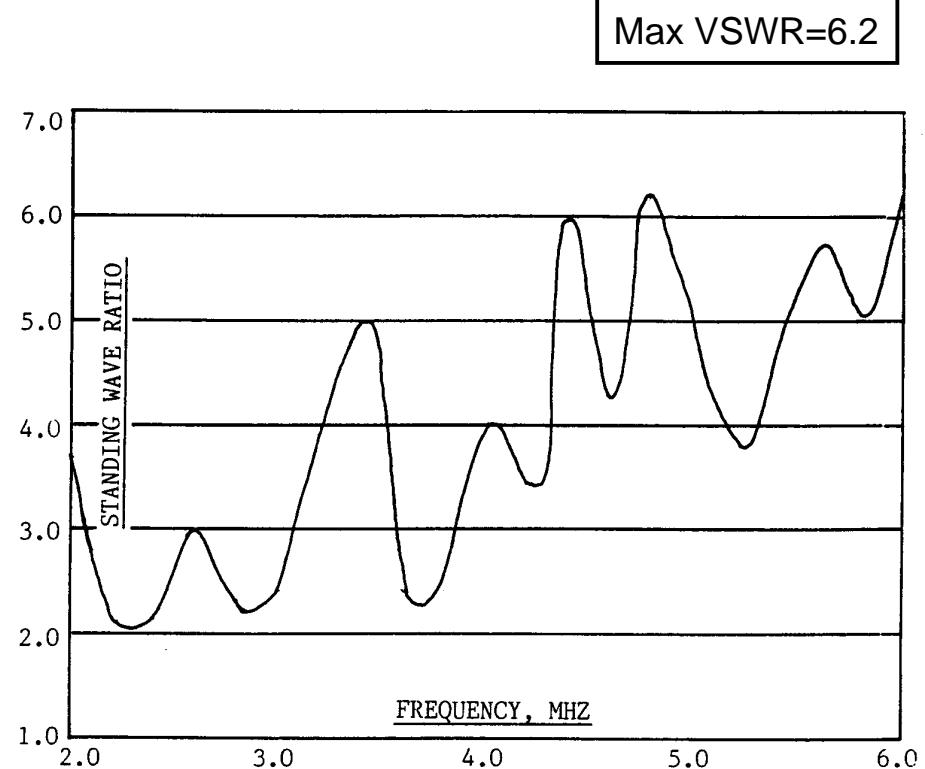
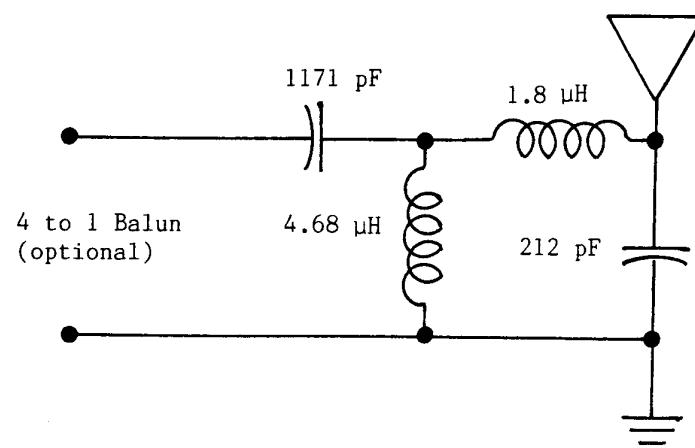


Caron's Example 11: Long Wire Receiving Antenna



Caron's Solution

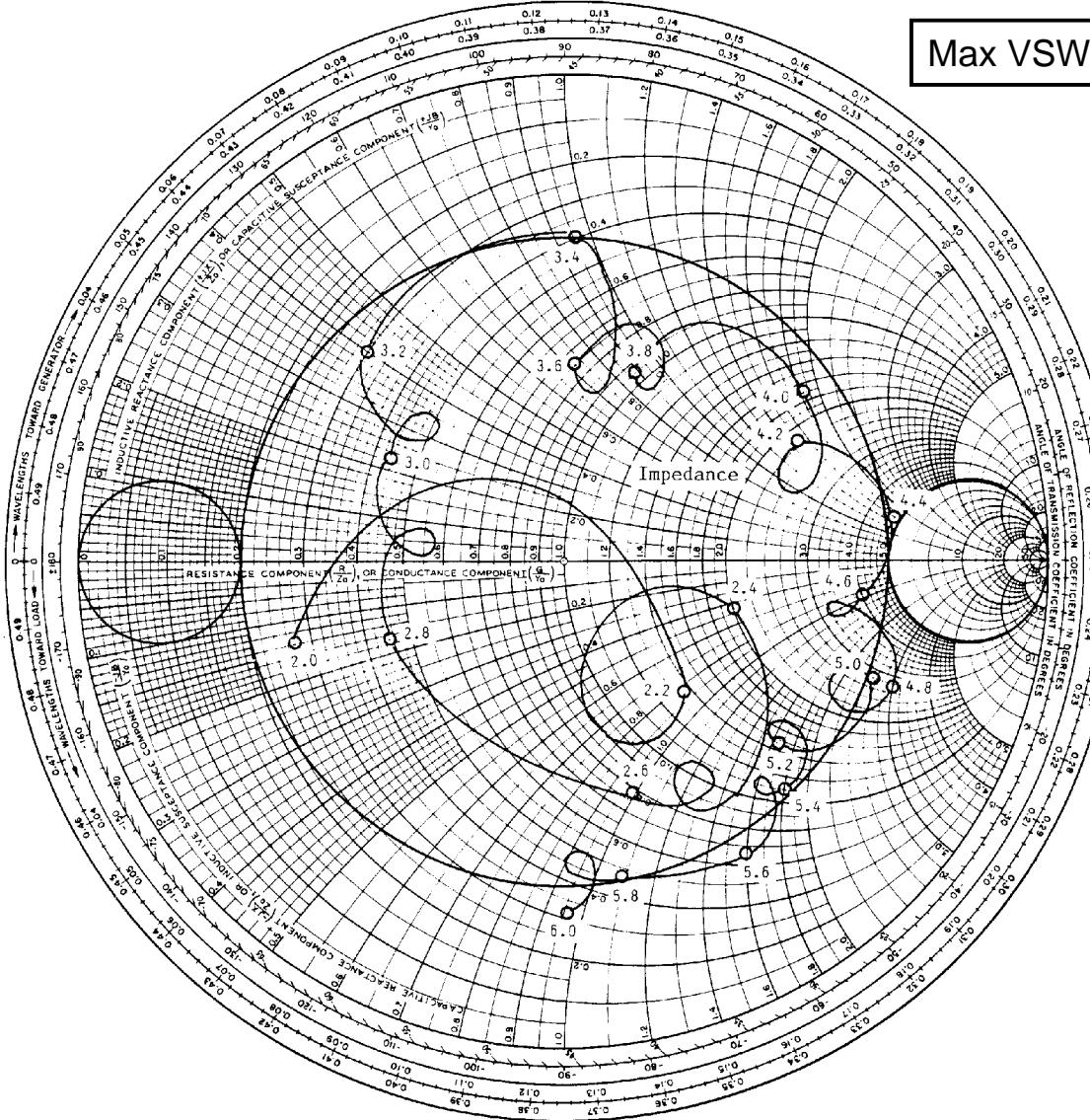
TRW



Caron's Solution



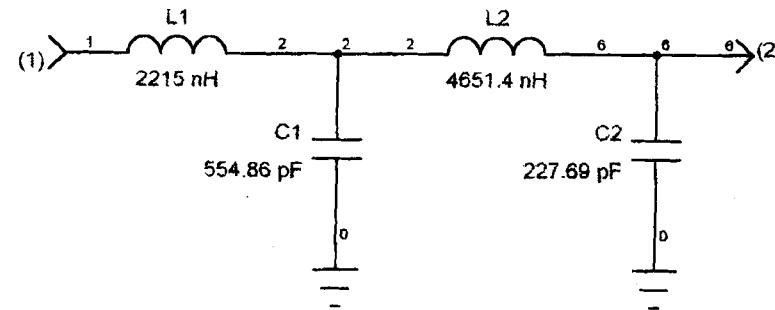
IMPEDANCE OR ADMITTANCE COORDINATES



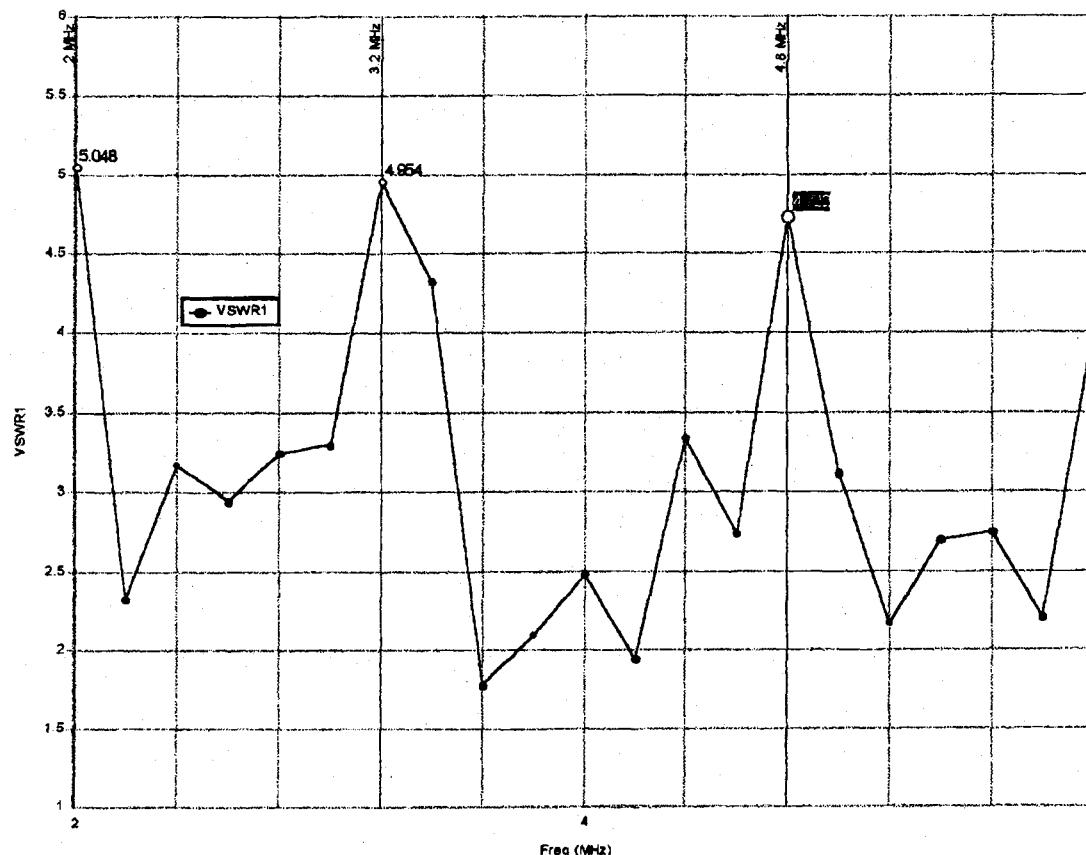
Max VSWR=6.2

Eagleware's Solution 1

TRW

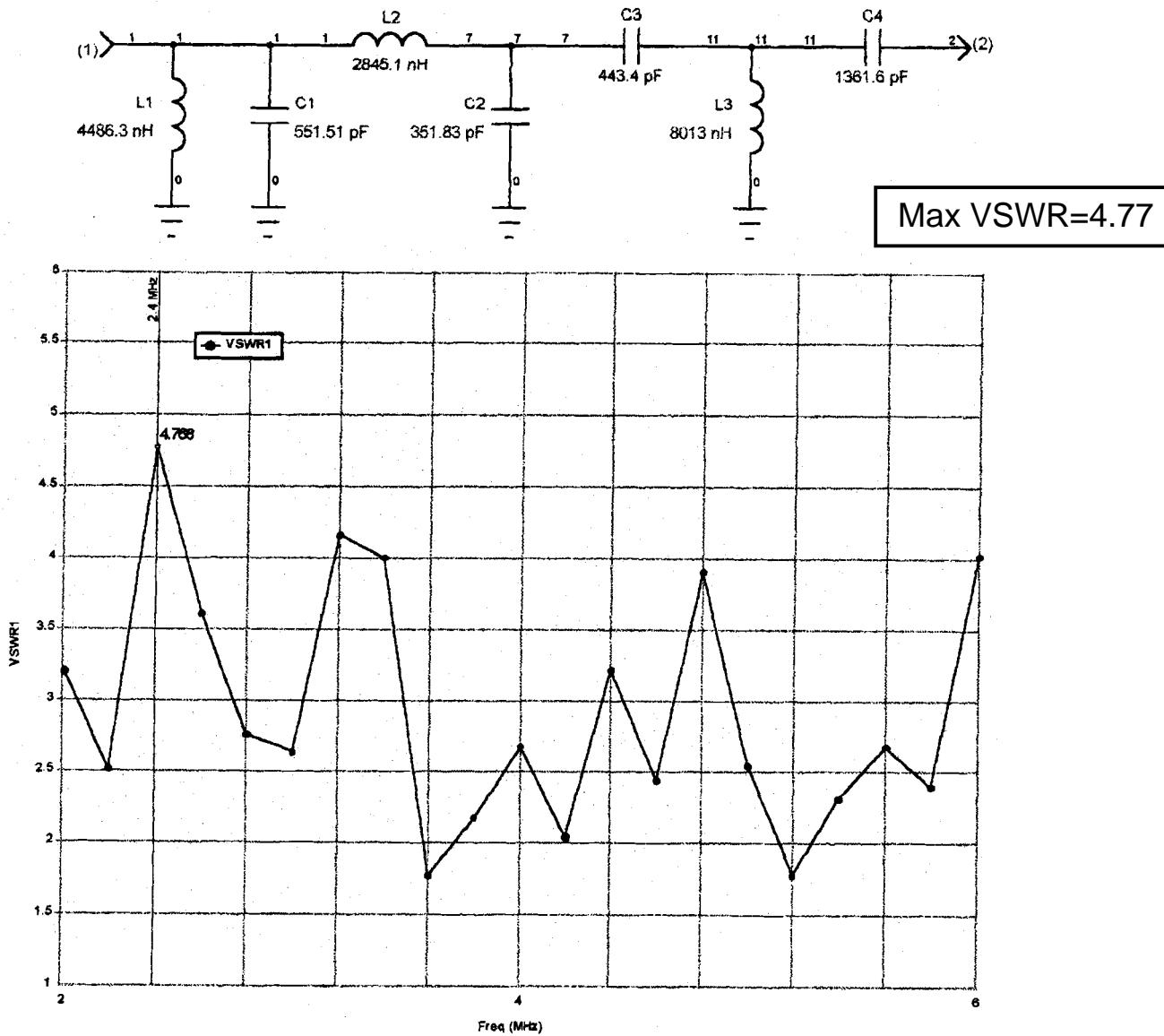


Max VSWR=5.05



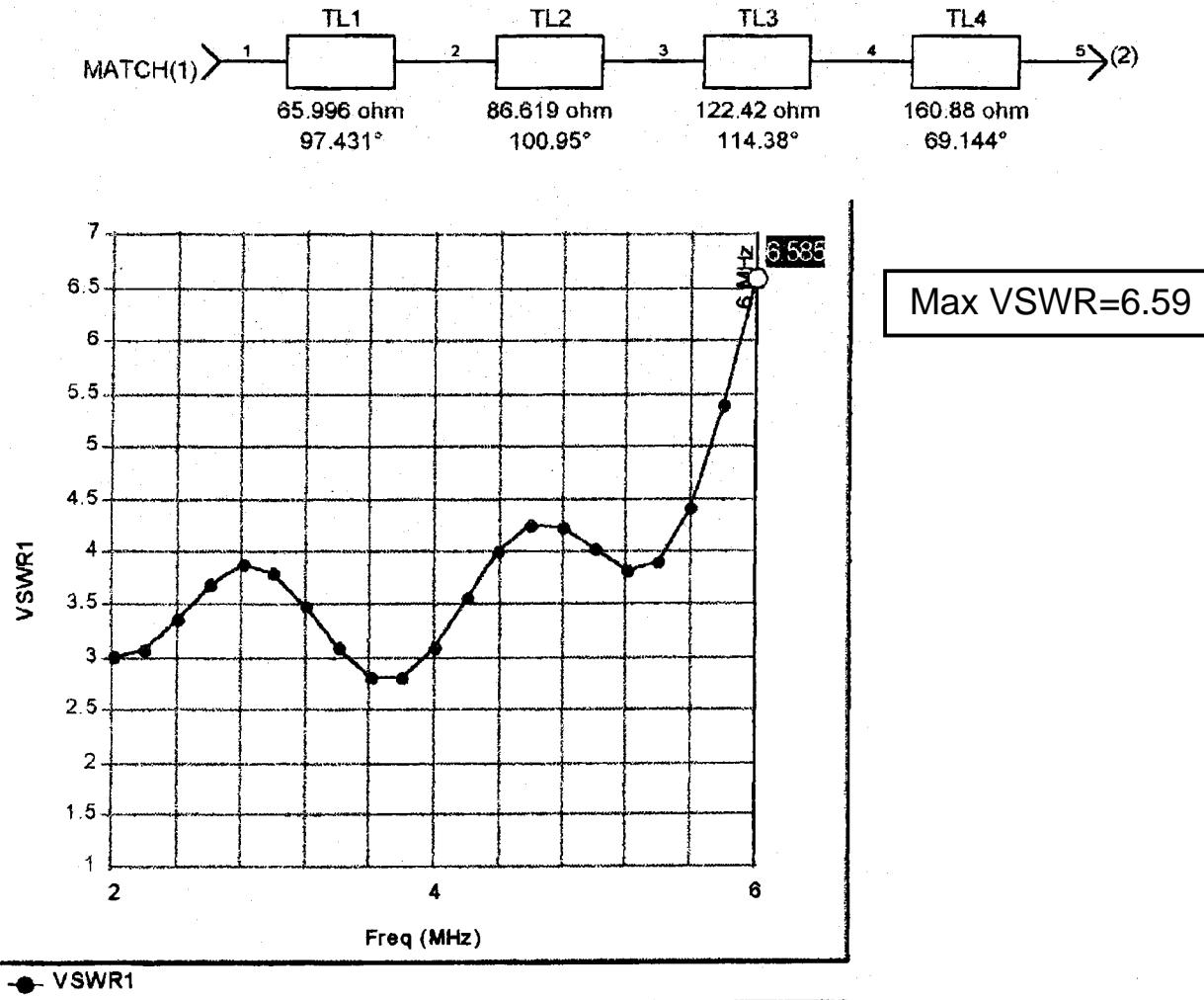
Eagleware's Solution 2

TRW

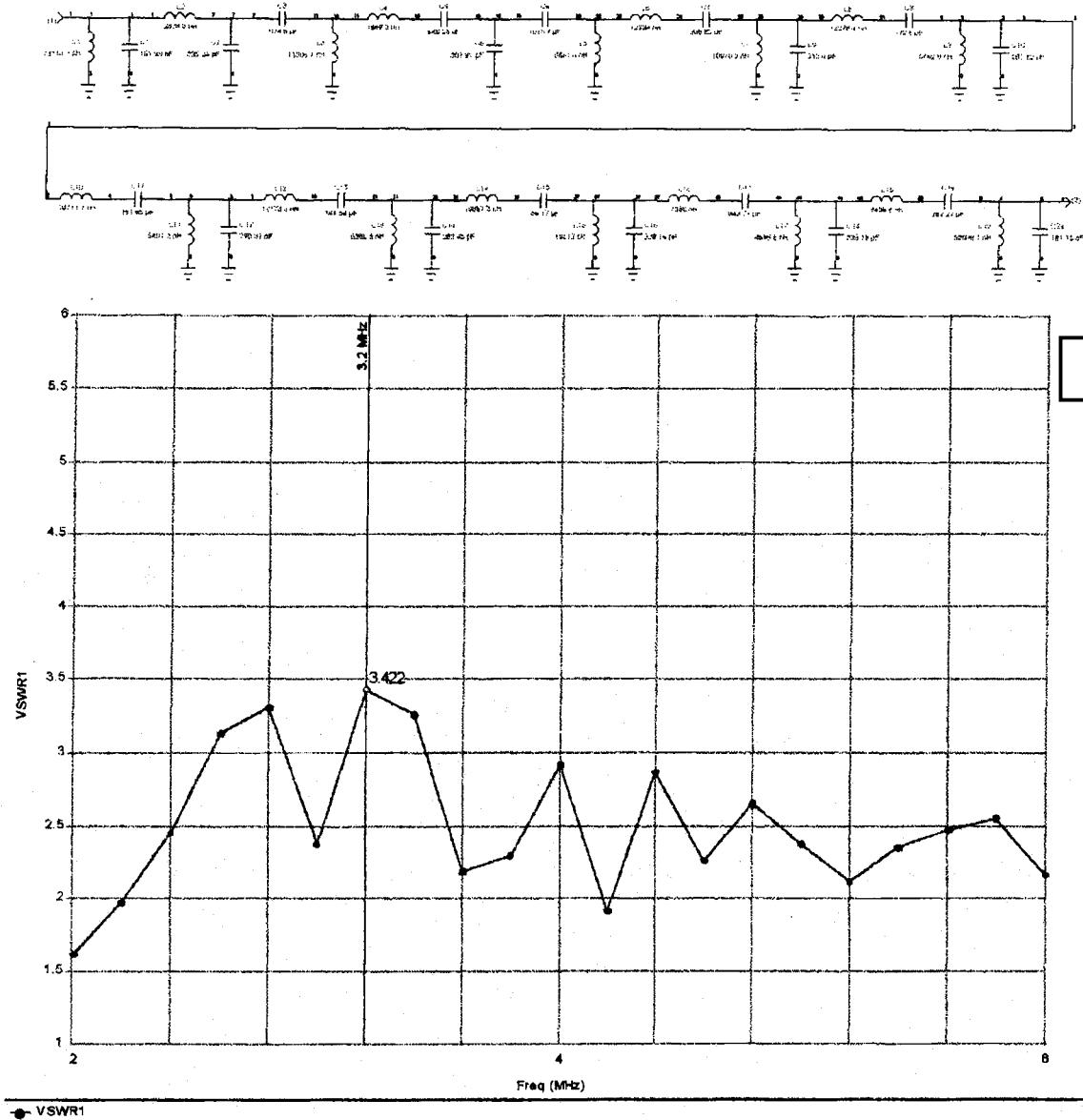


Eagleware's Solution 3

TRW

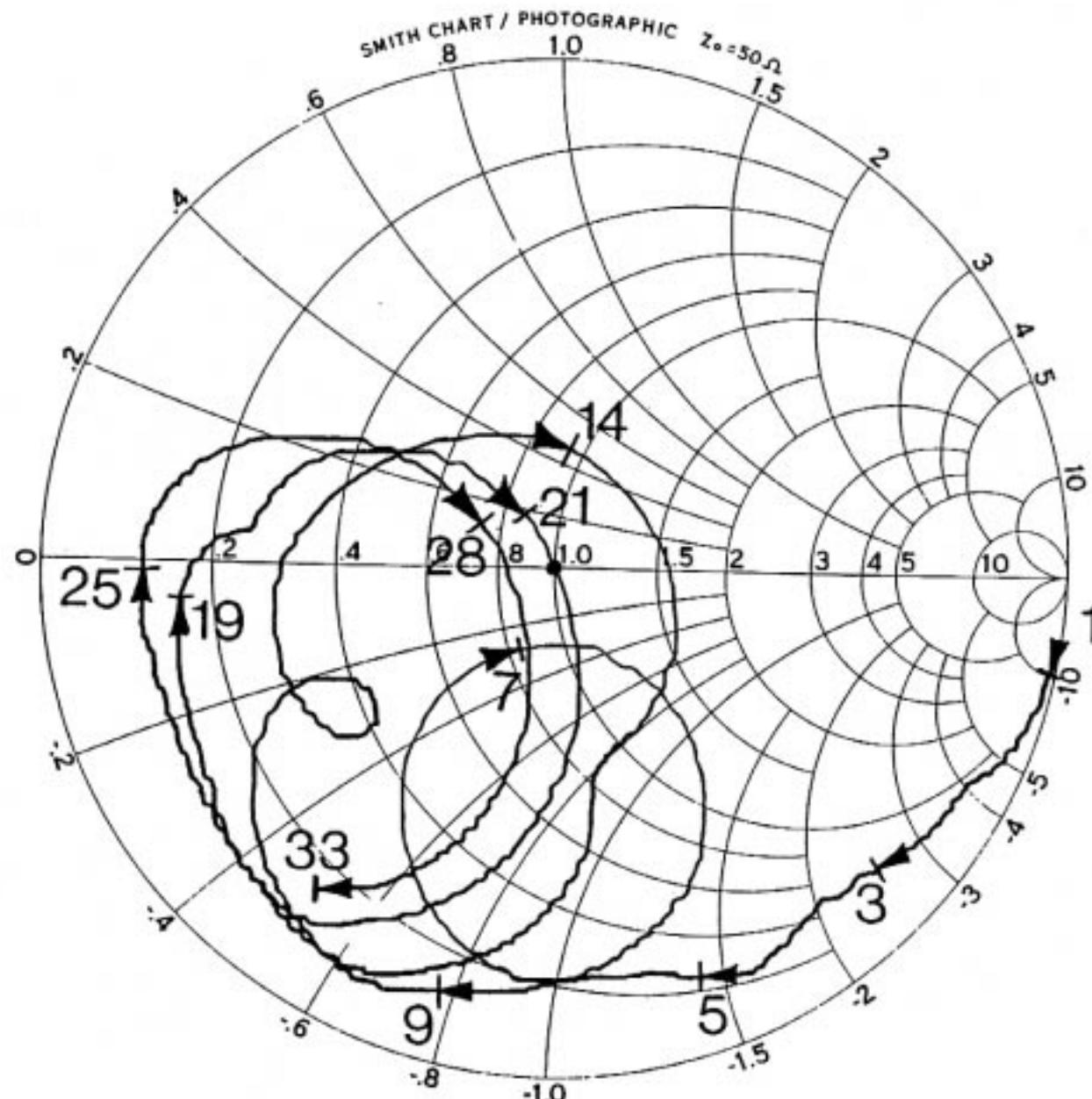


Eagleware's Solution 4: Fano Limit



GM3HAT 4-Band Dipole of Delight

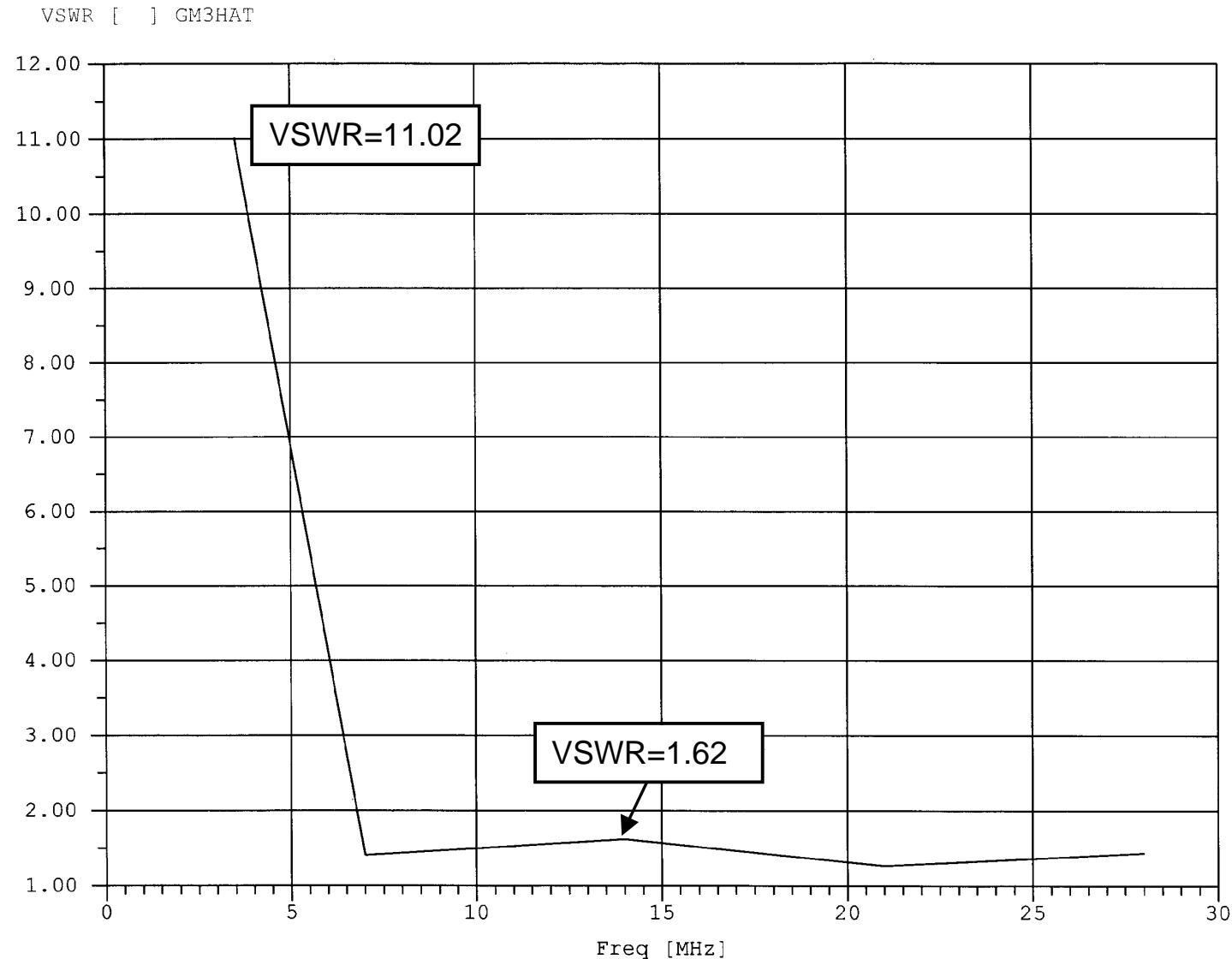
TRW



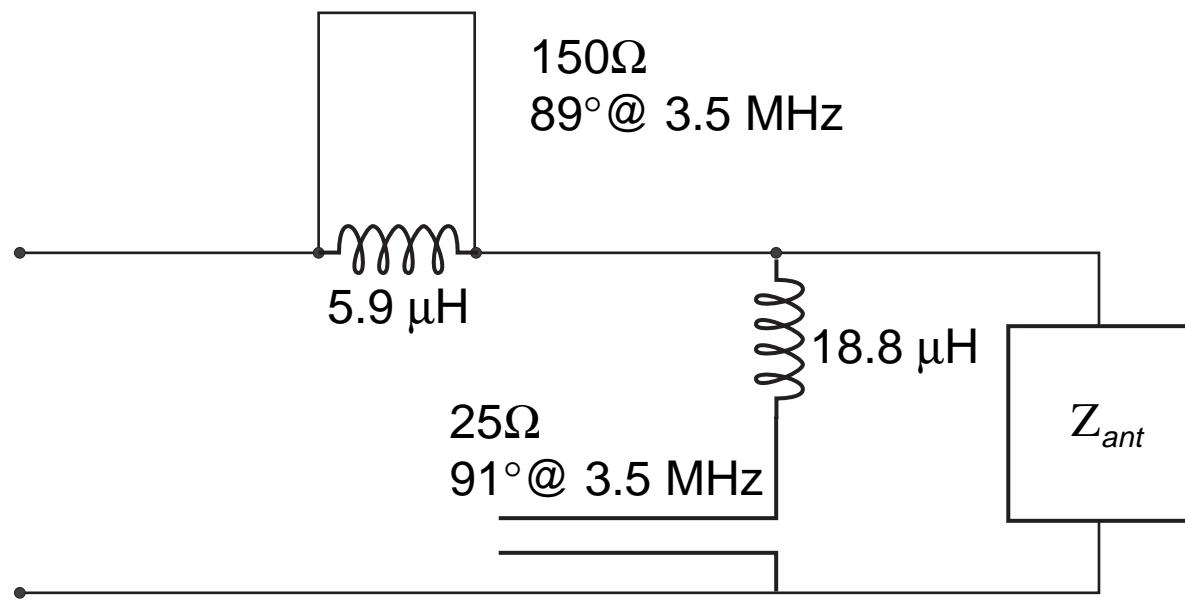
GM3HAT Feedpoint Impedance

At Five Harmonic Frequencies

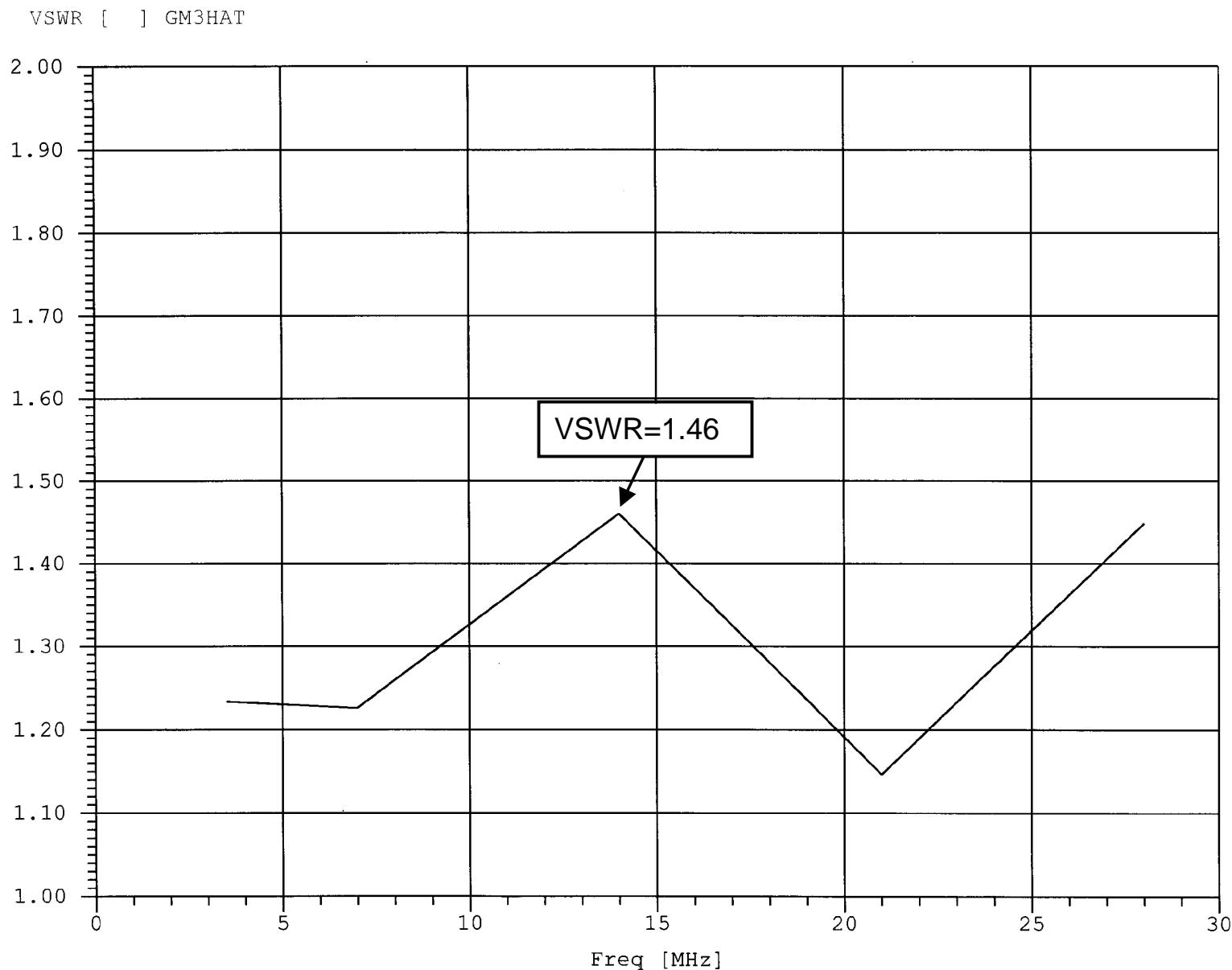
TRW



K6OIK 80m Suboctave Band Matching Network



K6OIK's Multiband Match Performance



□ Smith Chart Analysis & Display

- MicroSmith 2.3, ARRL, 1992, \$39**

A primitive DOS program.

- winSMITH 2.0, Noble Publishing, 1995, \$79**

Written by Eagleware. Easy to use. Restricted to ladder networks.
Doesn't have series stubs. Lacks an optimizer.

□ Matching Network Optimization & Synthesis

- ARRL Radio Designer 1.5, ARRL, 1995, \$150**

No longer sold. ARD's optimizer works with Serenade netlists and handles more variables than Serenade SV's optimizer.

- Serenade SV (student version), Ansoft, 2000, \$0 (free)**

Download (about 20 Mbytes) from:

<http://www.ansoft.com/about/academics/sersv/index.cfm>

- Advanced Automated Smith Chart 3.0, Artech House, 1998, \$395**

- =MATCH=, Eagleware, \$699 (requires GENESYS Basic, \$1997)**

- Harmonica Linear Design Suite, Ansoft, 2000, \$6900**

References: Articles



- Robert L. Thomas, “Broadband Impedance Matching in High-Q Networks,” *EDN*, pp. 62–69, December 20, 1973.
- Neal C. Silence, “The Smith Chart and Its Usage in RF Design,” *RF Design*, pp. 85–88, April 1992.
- Thomas R. Cuthbert, Jr., “Broadband Impedance Matching Methods,” *RF Design*, pp. 64–91, August 1994.
- Thomas R. Cuthbert, Jr., “Broadband Impedance Matching - Fast and Simple,” *RF Design*, pp. 38–50, November 1994.
- William E. Sabin, “Broadband HF Matching with ARRL Radio Designer,” *QST*, pp. 33–36, August 1995.
- William E. Sabin, “ARRL Radio Designer and the Circles Utility, Part 1: Smith Chart Basics,” *QEX*, pp. 3–9, Sept/Oct 1998.
- William E. Sabin, “ARRL Radio Designer and the Circles Utility, Part 2: Small-Signal Amplifier Design,” *QEX*, pp. 3–11, Nov/Dec 1998.

References: Articles (Cont'd)



- Steve Sparks, “A Practical Amateur Application of the Smith Chart,” *Communications Quarterly*, pp. 102–106, Summer 1999.
 - *This article contains serious Smith charting errors. See the comments by Garry Shapiro, NI6T, Communications Quarterly, p. 3, Fall 1999.*
- K.C. Chan and A. Harter, “Impedance Matching and the Smith Chart – The Fundamentals,” *RF Design*, pp. 52–66, July 2000.

References: Books



- Robert A. Chipman, *Transmission Lines*, Schaum Outline Series, McGraw-Hill, 1968.
 - Basic, mathematical. A classic, but out-of-print.
- Robert L. Thomas, *A Practical Introduction to Impedance Matching*, Artech House, 1976, ISBN 0890060509.
 - Intermediate, mathematical. A nice treatment of 4-element networks for wideband matching.
- Pieter L. D. Abrie, *The Design of Impedance-Matching Networks for Radio-Frequency and Microwave Amplifiers*, Artech House, 1985, ISBN 0890061726.
 - Advanced, mathematical.
- Wilfred Caron, *Antenna Impedance Matching*, ARRL, 1989, ISBN 0872592200.
 - Intermediate, non-mathematical. Caron illustrates manual Smith chart methods at their best, but which nonetheless have been completely replaced by computer-aided network design.
- Brian C. Wadell, *Transmission Line Design Handbook*, Artech House, 1991, ISBN 0890064369.
 - Analysis and formulas for many transmission lines. Comparable to M.A.R. Gunston or K.C. Gupta, et al., below.

References: Books (Cont'd)



- Phillip H. Smith, *Electronic Applications of the Smith Chart in Waveguide Circuit, and Component Analysis*, 2nd edition, Noble Publishing, 1995, ISBN 1884932398.
 - Originally published by McGraw-Hill, 1969, and reprinted by Krieger, 1983. Intermediate, mathematical.
- K.C. Gupta, R. Garg, I. Bahl, and P. Bhartia, *Microstrip Lines and Slotlines*, 2nd ed., Artech House, 1996, ISBN 089006766X.
 - Analysis and formulas for many transmission lines.
- M.A.R. Gunston, *Microwave Transmission-Line Impedance Data*, Noble Publishing, 1997, ISBN 1884932576.
 - Originally published by Van Nostrand Reinhold, 1972. Analysis and formulas for many transmission lines. Comparable to B. Wadell above.
- ARRL Antenna Book, 18th edition, chapters 24-28, ARRL, 1997, ISBN 0872596133.
 - Elementary, non-mathematical.
- M. Walter Maxwell, W2DU, *Reflections II: Transmission Lines and Antennas*, 2nd ed., Worldradio Books, 2001, ISBN 0970520603.
 - A non-mathematical treatment of transmission lines and matching that examines and corrects common misunderstandings.

References: Application Notes, Videos, and Web Sites



❑ Application Notes

- Times Microwave's *Complete Coaxial Cable Catalog & Handbook*
Download from:
<http://www.timesmicrowave.com/products/military/TL14/TL14.htm>

❑ Videos

- Glenn Parker, *Introduction to the Smith Chart*, Noble Publishing, 1996. 50 minutes, \$99.

❑ Useful Web Sites

- <http://www.timesmicrowave.com/calculators/index.htm>
- <http://www.sss-mag.com smith.html>
- <http://www.ee.surrey.ac.uk/Personal/D.Jefferiessmith.html>
- <http://www.scott-inc.com/htmlsmith.htm>

❑ This Presentation is at

- <http://www.fars.k6ya.org/docssmithchart.html>